Optimization and Inference for Cyber-Security in Complex Engineered Networks

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National Taiwan University

Motivation

Rumors

成龍 Jackie Chan

Jackie is alive and well. He did not suffer a heart attack and die, as was reported on many social networking sites and in online news reports.

Jackie is fine and is busy preparing for the filming of his next movie.

Yesterday at 8:30am · Share

45,971 people like this.

View all 6,721 comments

@petershankman
Peter Shankman

Dear CNN: Morgan Freeman is still busy living. He's yet to get busy dying. Please confirm first.

15 hours ago via ÜberTwitter · Favorite · Retweet · Reply

Peter Shankman, Twitter

Source: Google

Motivation

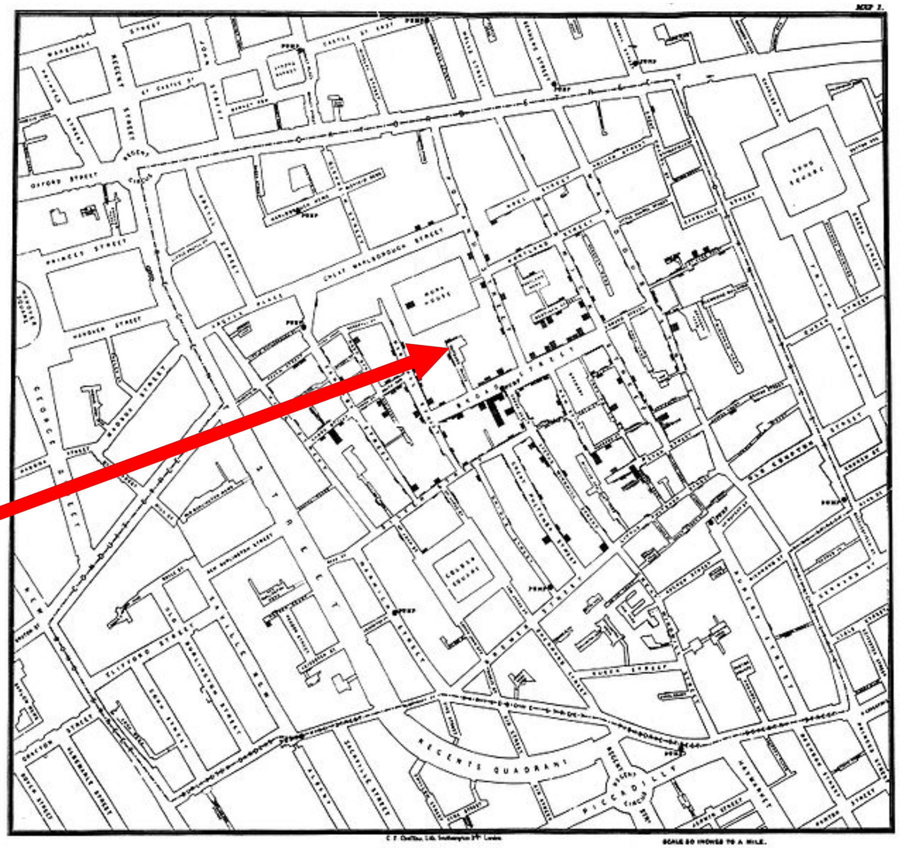
Rumors

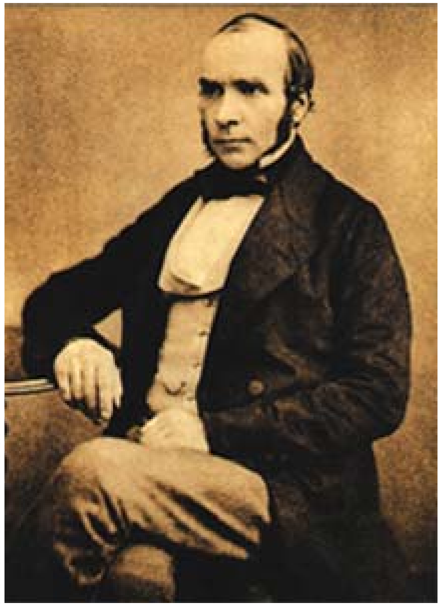
Source: Bloomberg

Motivation

1854 London Cholera Epidemic

Outbreak source



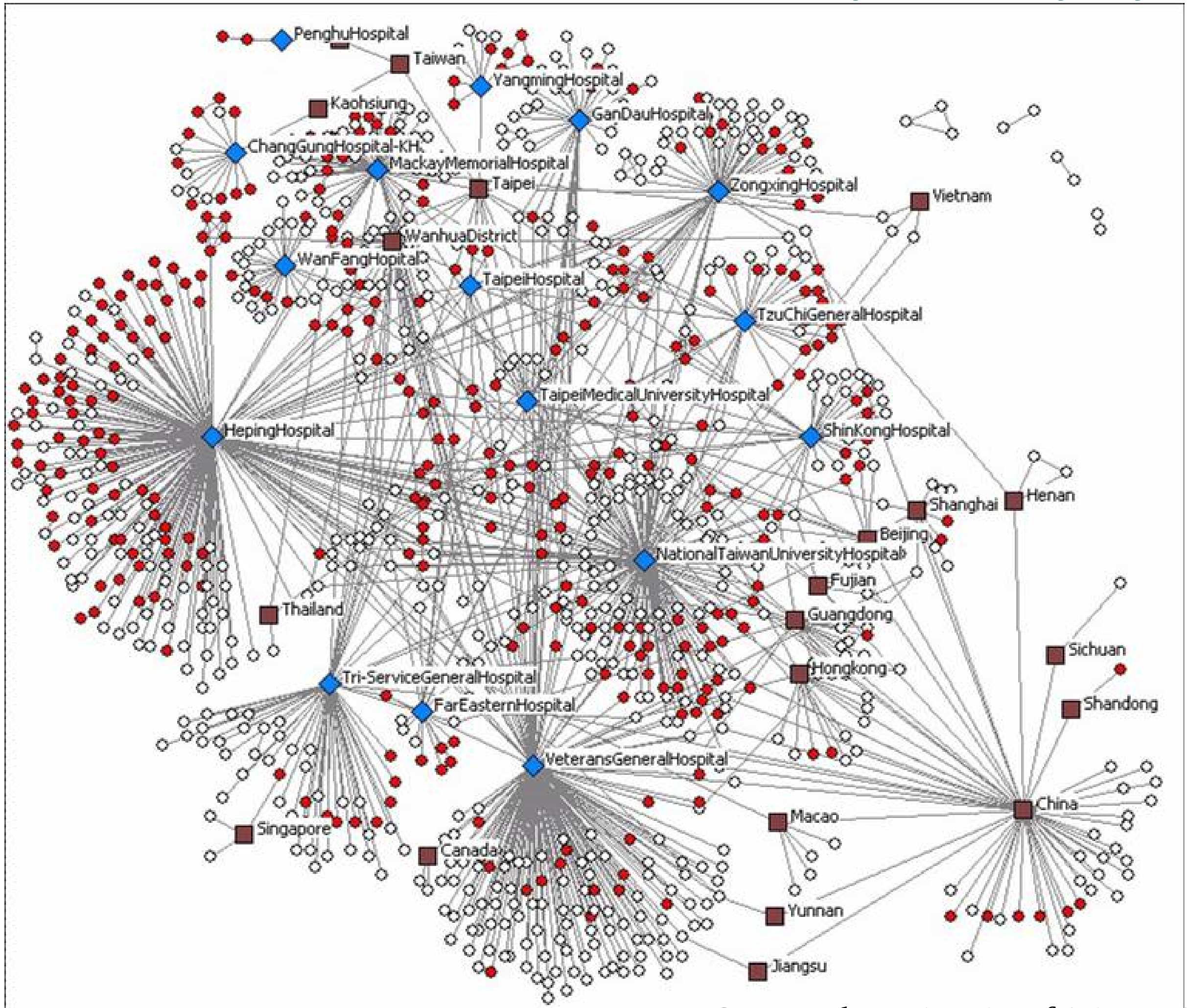


Dr. John Snow

Source: Wikipedia

Motivation

2003 SARS (severe acute respiratory syndrome)



Source: The University of Arizona Artificial Intelligence Lab

Motivation
2003 SARS Rumor

Epicenter of Hong Kong SARS

329 Infected
42 killed

- Rumor: "Entire City was poised to be quarantined"
- 14 year-old boy arrested for creating fake news page

Source: http://edition.cnn.com/2013/02/21/world/asia/hong-kong-sars-anniversary

Rumor and panic spread faster than virus. Nothing spreads like fear!

Motivation

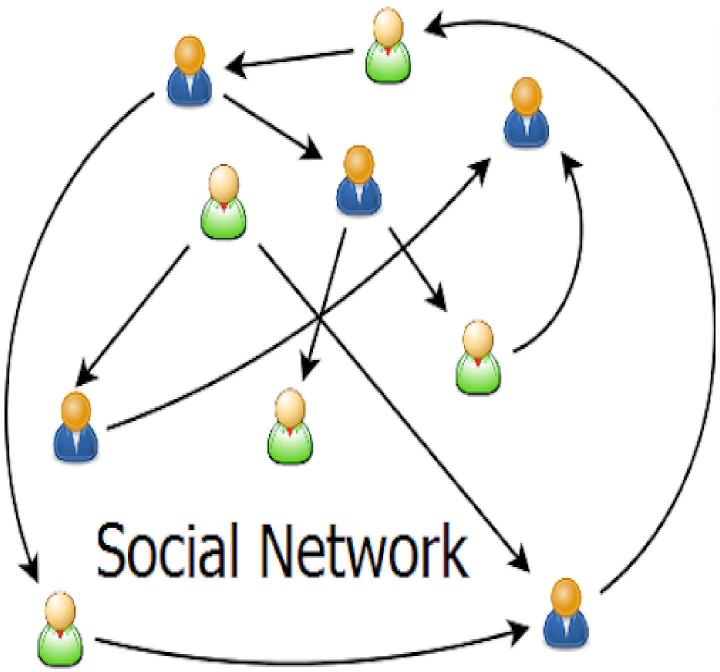
2009 Online Flu Map Goes Viral

- Created by Pittsburgh biochemist Harry Niman on April 21, 2009
- Surpassing 290,000 web views and 3000 comments within 9 days

Source: http://edition.cnn.com/2009/TECH/04/30/online.flumaps/index.html

Motivation

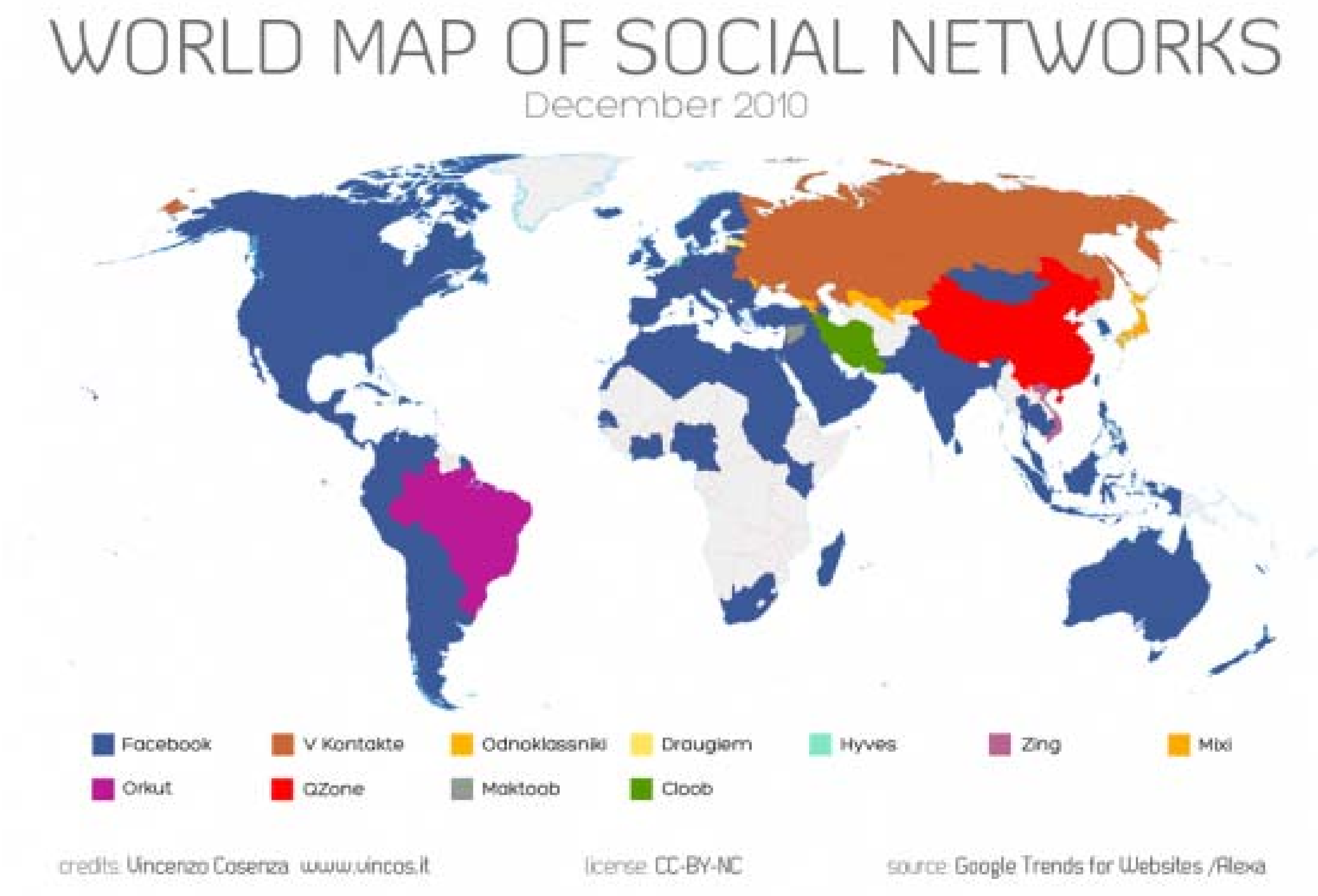
Growth and Expansion of Online Social Networks



- Smartphone revolution 2007
- Twitter 2006
- Tencent Weixin 2012

Motivation

Reach of Online Social Networks



Motivation
National Grand Challenges

NAE GRAND CHALLENGES FOR ENGINEERING

The usefulness of these approaches depends on numerous variables — how infectious and how deadly the virus is, the availability of antiviral drugs and vaccines, and the degree of public compliance with quarantines or travel restrictions. Again, understanding the mathematics of networks will come into play, as response systems must take into account how people interact. Such models may have to consider the “small world” phenomenon, in which interpersonal connections are distributed in a way that assists rapid transmission of the virus through a population, just as people in distant parts of the world are linked by just a few intermediate friends.

– US National Academy of Engineering Grand Challenges 2008

– Challenge No. 7: Advance health informatics

Motivation
National Grand Challenges

NAE GRAND CHALLENGES FOR ENGINEERING

All engineering approaches to achieving security must be accompanied by methods of monitoring and quickly detecting any security compromises. And then once problems are detected, technologies for taking countermeasures and for repair and recovery must be in place as well. Part of that process should be new forensics for finding and catching criminals who commit cybercrime or cyberterrorism.

– US National Academy of Engineering Grand Challenges 2008

– Challenge No. 11: Secure Cyberspace

Motivation

DARPA Mathematical Challenges

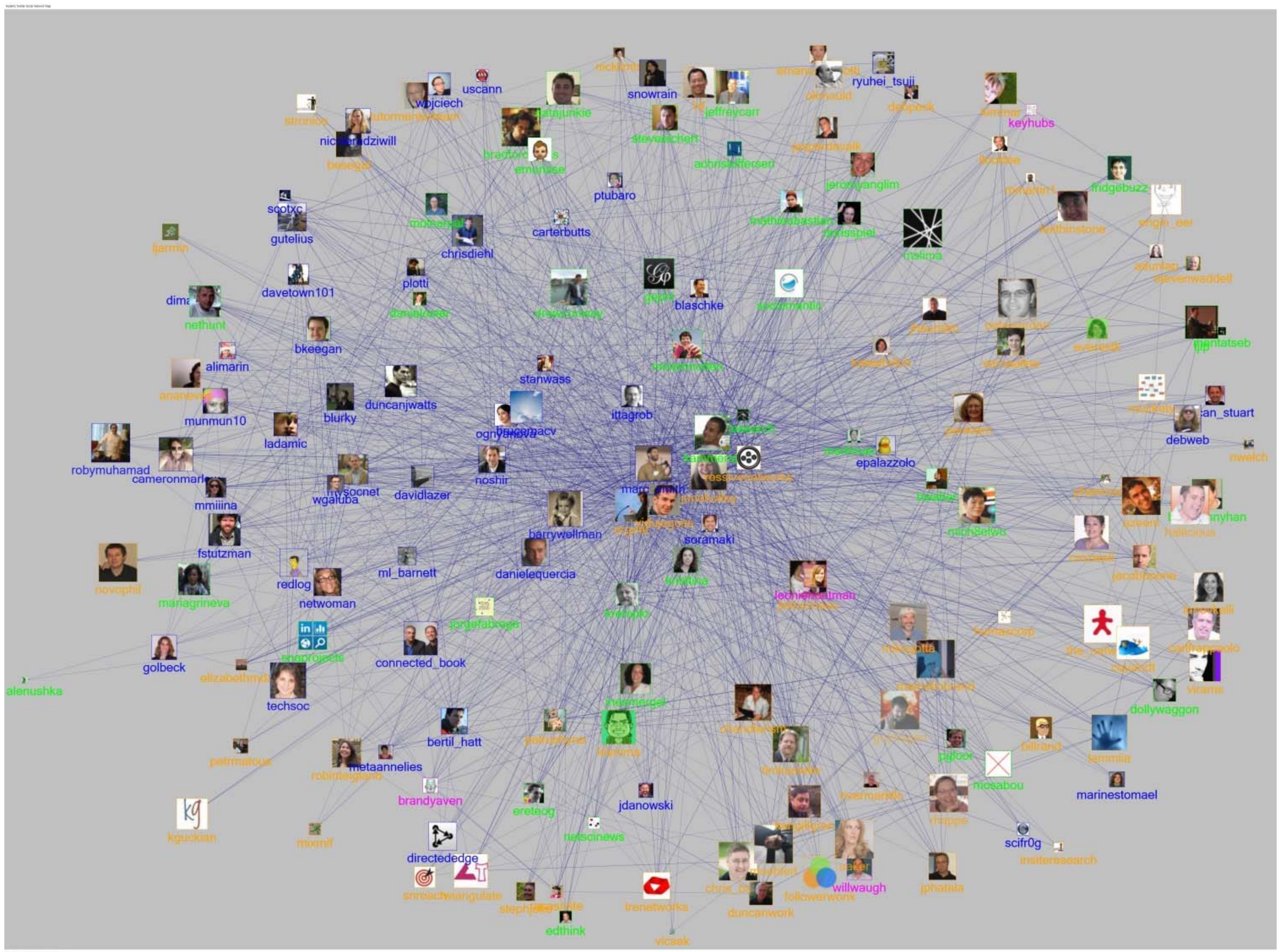
- 23 Mathematical Challenges issued by US Defense Advanced Research Projects Agency (DARPA) in 2008

- Challenge No. 2: The Dynamics of Networks
- Challenge No. 14: An Information Theory for Virus Evolution

  - **can Shannon's theory shed light on this fundamental area**?

Source: https://www.fbo.gov/download/9bc/9bce380aafb19f9ad3bda188bfc1ab20/DARPA-BAA-08-65.doc

Rumor Spreading in Online Social Network

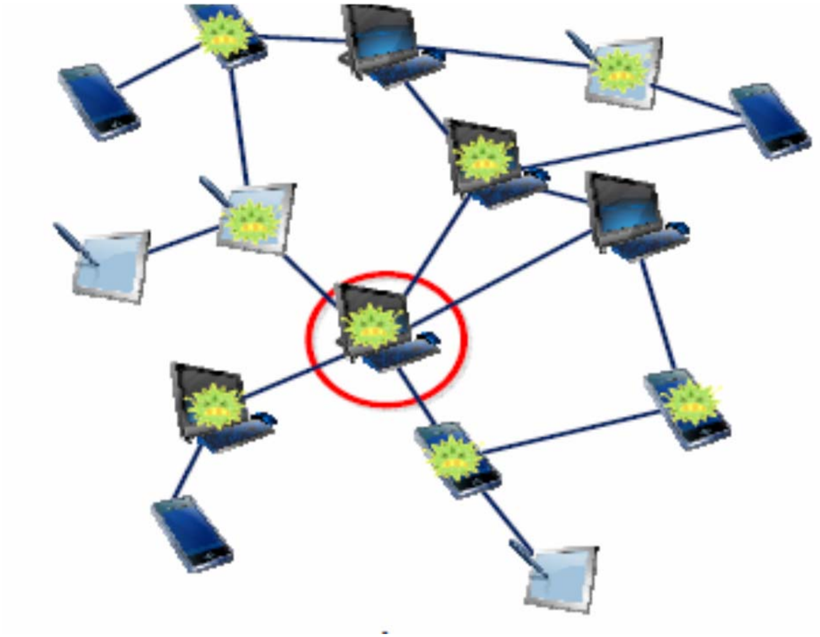


- Outbreak of infectious virus
- Diffusion of viral Information in Network
- Cause of outbreak

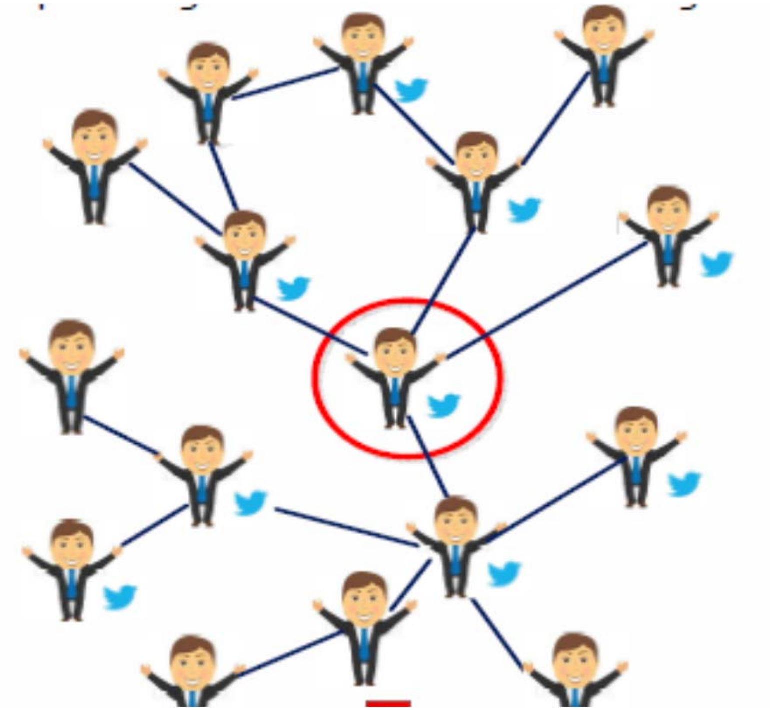
- Epidemic-like information flow = rumor spreading in a network.

Who is the culprit?

- **Spread of computer virus**



- **Tweeting and Retweeting in Twitter Network**



- A rumor, originating from a suspect set, spreads on a network.
- We only know the prior suspect set and infected nodes.
- Can we find the single rumor source?

Literature

----Research on epidemic outbreak/rumor spreading

- understand impacts of network structure and infection/cure rates

  [Moore—PRE’00, Pastor-Satorras—PRL’01, Newman—PRE’02]

- learn network parameters and predict propagation characteristics

  [Streftaris—IWSM’02, Okamura—ISSRE’07, Gomez-Rodriguez—SIGKDD’10]

- extract influential source nodes

  [Kempe—SIGKDD’03, Chen—SIGKDD’09, Dong—Allerton’12]

– **Rumor source estimation problem has only been recently studied.**

Literature

----estimation of rumor source

- identification of single rumor source using SI model

  [Shah—TIT’11, Shah—SIGMETRICS’12]

- geometric trees, random graphs

  [Shah—arXiv’11, Shah—TIT’11]

- identification of multiple rumor sources (SI model), identification of single rumor source (SIR model)

  [Luo—TSP’13, Zhu—ITA’13]

- noisy estimation of a single rumor source

  [Pinto—PRL’12]

– **Important features such as suspects, no. of observations, topology has not been considered.**

SI Spreading Model
(Kermack & McKendrick 1927)

- **Fixed Population *N (only one infected at each* t*)***
- **Susceptible Set at time *t*** $S(t)$
- **Infected Set at time *t*** $I(t)$

S_t denote $|S(t)|$ and I_t denote $|I(t)|$

$$S_t + 1 = S_t + 1$$

$$I_t + 1 = I_t + 1$$

$$S_0 = N, I_0 = 0.$$

SI Spreading Model

- **Vertex of a graph G to model the susceptible and the infected node (person)**
- **An edge in G models the relationship between two nodes**
  - **Two persons connected as Facebook Friends or Twitter Follower**

Let G_t be a subgraph of order t of G.

G_t is composed of t infected vertices

G_1 rumor source

$|G_t + 1| = |G_t| + 1$

Random SI Model for Rumor Spreading

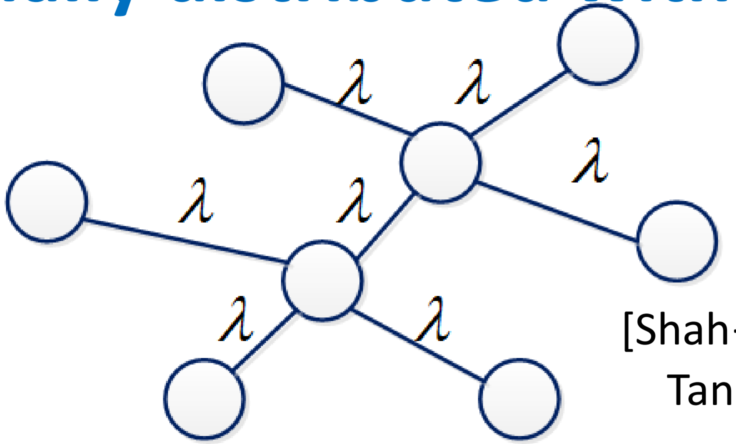
■ SI (susceptible-infectious) model

- An infected node keeps the rumor forever

■ Uniform probability of any node in a prior suspect set being source

- $P_s(\text{source} = s) = |S|^{-1}, s \in S$ ➡ S consists of suspect nodes

■ Time to infect neighbor is independent and exponentially distributed with rate λ



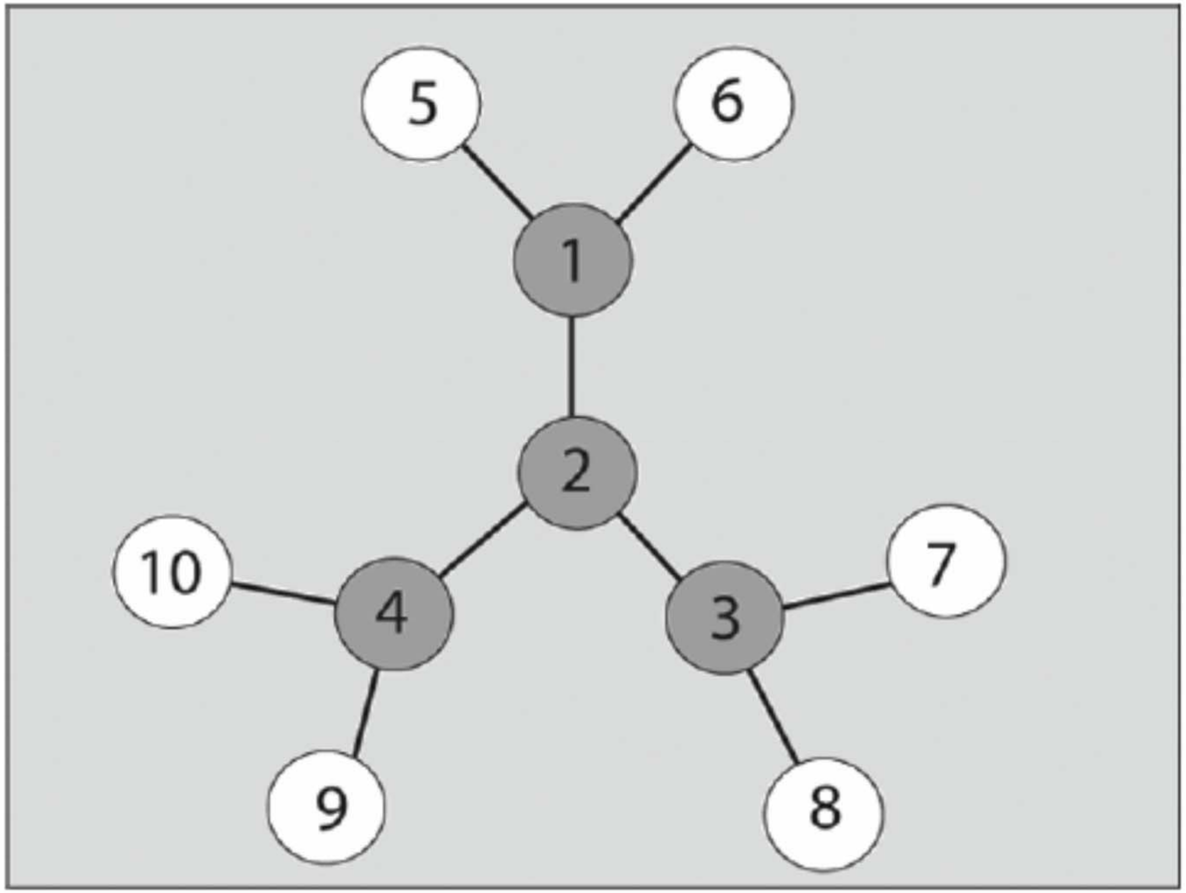
[Shah—TIT'11, Shah—SIGMETRICS'12]
Tan—ISIT'13, SIGMETRICS'14]

Deterministic SI Model

- **Time to infect neighbor is assumed to be a fixed time-slot**
- **A susceptible node is infected by each of its neighbors with probability *q***
- **Assuming the number of infected neighbors of a susceptible node is *n*, the probability that the node becomes infected is** $1-(1-q)^n$
- **Time-slot model used by researchers for network inference** [Luo—TSP'13, Zhu—ITA'13]

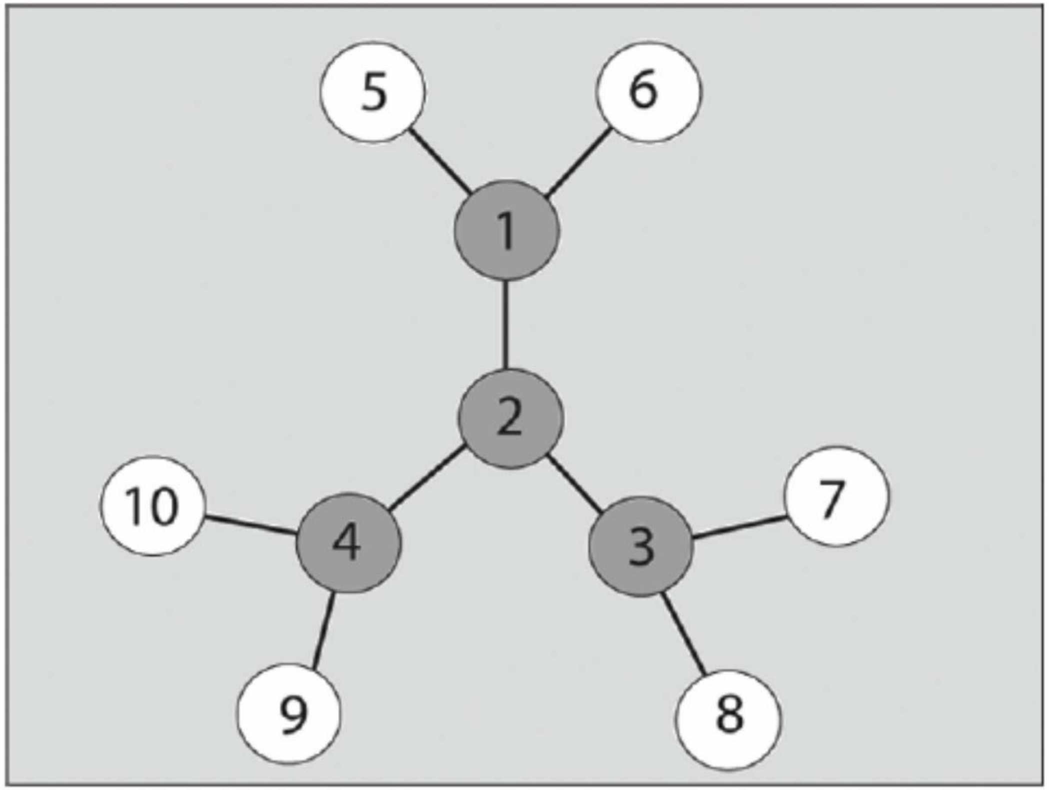
Inference with Single Snapshot Observation

Toy Example



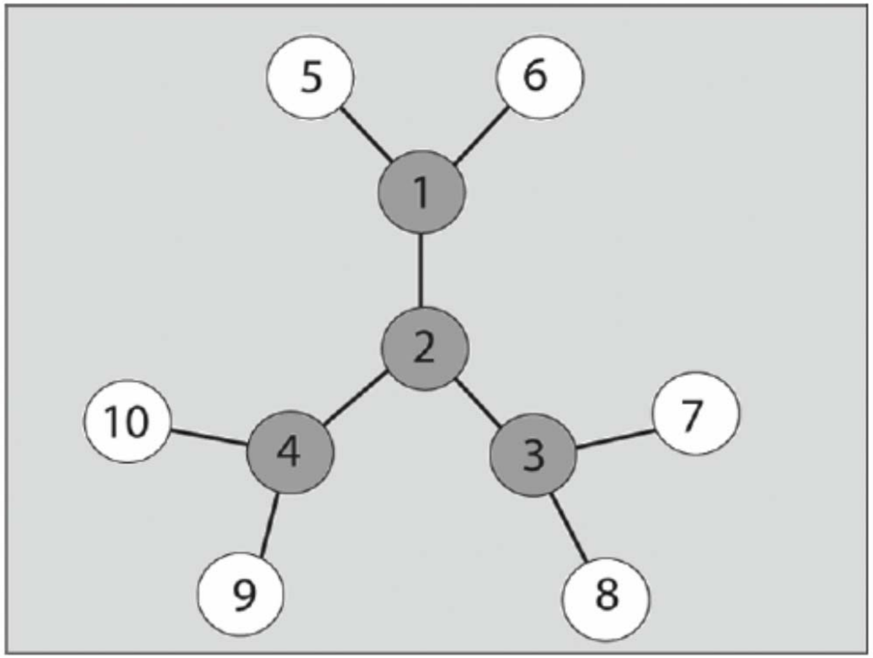
Toy Example

- **Counts the number of permitted permutation to spread a rumor**



Toy Example

- **Suppose Source 1**
  - **Two permutations: {1,2,3,4}, {1,2,4,3}**
- **Suppose Source 2**
  - **Six permutations: {2,1,3,4}, {2,1,4,3},{2,3,1,4}, …**



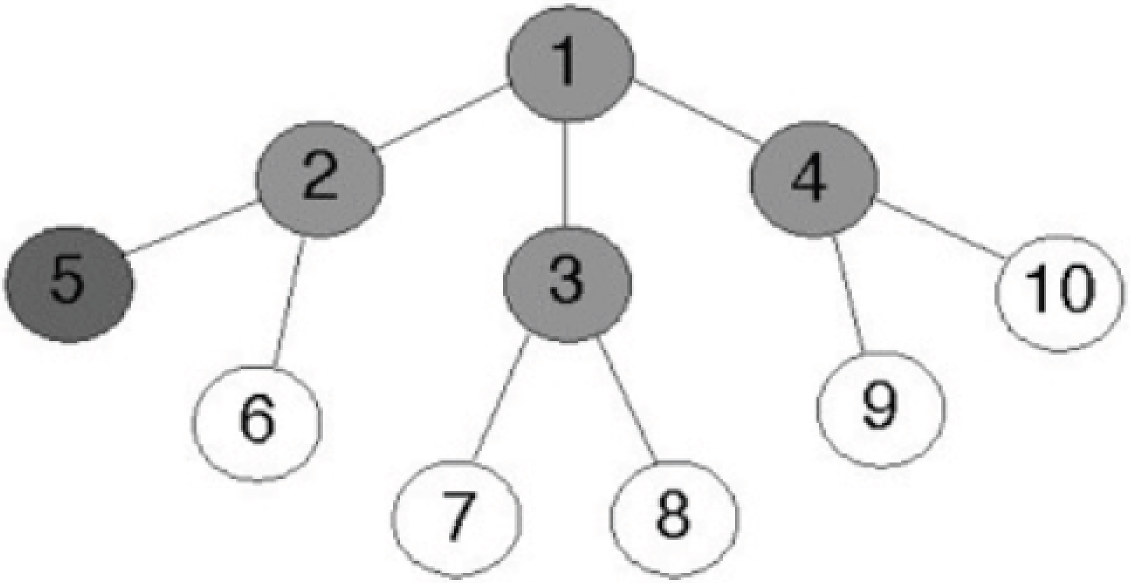
[Shah—TIT'11]

Inference in Tree

- Let T be a tree :
- let G_n be the subtree of T at time n
- $P(G_n|v^*)$ is the probability that view v^* as the source
- Let σ_i be the possible infecting order
- $S(v^*, G_n)$ be the collection of all σ_i where v^* is viewed as the source

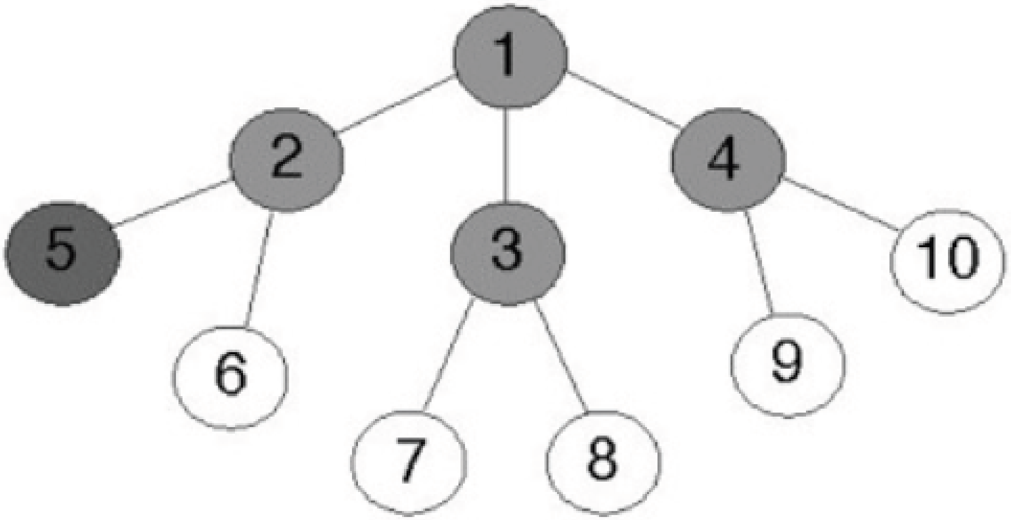
$$P(G_n|v^*) = \sum_{\sigma_i \in S(v^*, G_n)} P(\sigma_i|v^*).$$

Toy Example



Toy Example

Suppose Node 1 is Rumor

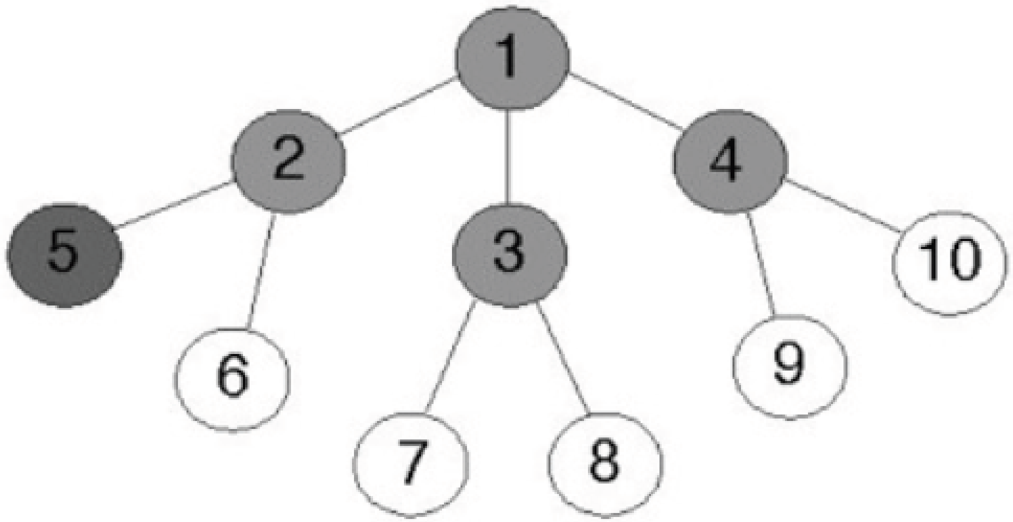


$\sigma_1 = v_1 \to v_2 \to v_3 \to v_4$ $\sigma_2 = v_1 \to v_2 \to v_4 \to v_3$ $\sigma_3 = v_1 \to v_3 \to v_2 \to v_4$

$\sigma_4 = v_1 \to v_3 \to v_4 \to v_2$ $\sigma_5 = v_1 \to v_4 \to v_3 \to v_2$ $\sigma_6 = v_1 \to v_4 \to v_2 \to v_3$

Toy Example

Suppose Node 1 is Rumor

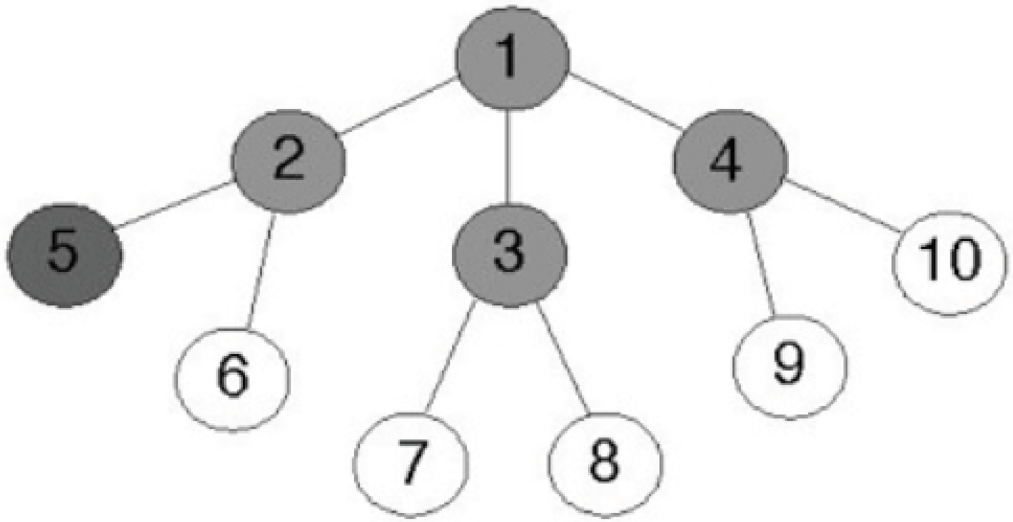


Let's calculate the probability of σ_1

$$P(\sigma_1 | v_1) = \frac{1}{3} \cdot \frac{1}{4} \cdot \frac{1}{5}$$

Toy Example

Suppose Node 1 is Rumor



$$P(\sigma_i | v_1) = \frac{1}{3} \cdot \frac{1}{4} \cdot \frac{1}{5} \quad \text{for } i = 1, 2, 3, 4, 5, 6$$

General Tree

$$P(\sigma_i | v_1) = \prod_{k=1}^{n-1} \frac{1}{\sum_{v_i \in V(G_k)} d(v_i) - 2(k-1)}$$

where G_k is a subgraph of G_n and it represents the infected subgraph at k_{th} time step along with the infecting order σ_i .

General Degree-Regular Tree

$$P(\sigma_i|v_1) = \prod_{k=1}^{n-1} \frac{1}{\sum_{v_i \in V(G_k)} d(v_i) - 2(k-1)}$$

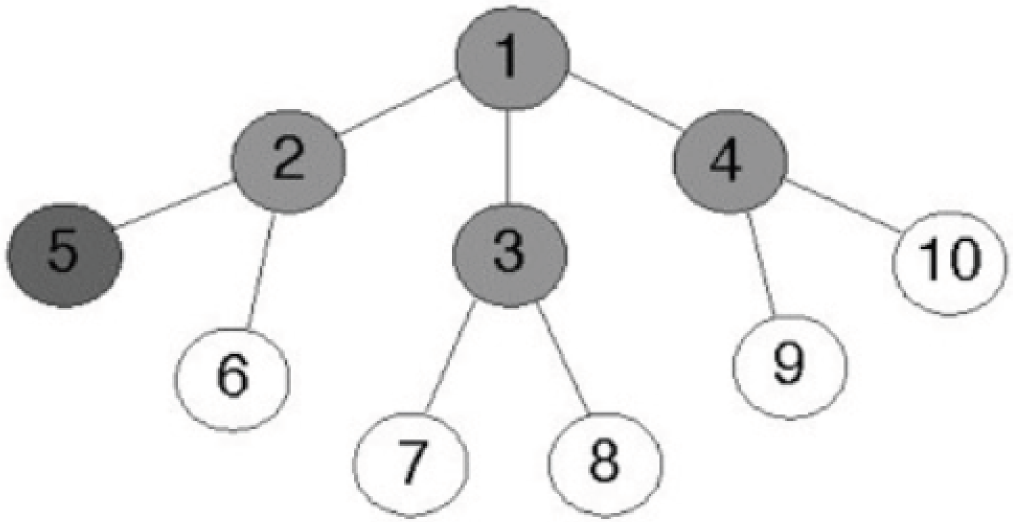
where G_k is a subgraph of G_n and it represents the infected subgraph at k_{th} time step along with the infecting order σ_i .

$$\text{For } d - regular \text{ tree}$$

$$P(\sigma_i|v_1) = \prod_{k=1}^{n-1} \frac{1}{dk - 2(k-1)}$$

Toy Example

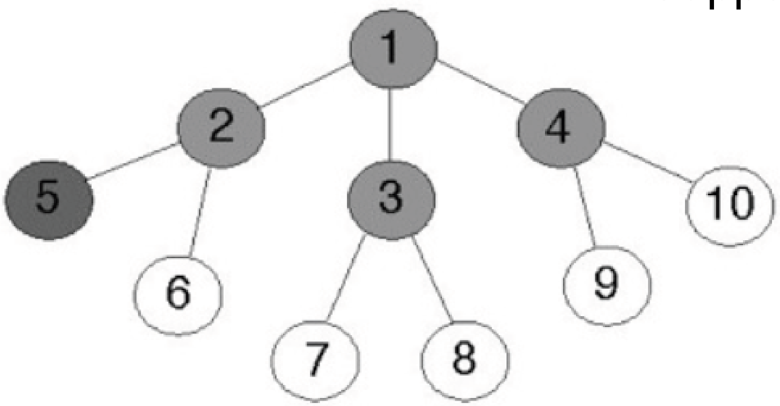
Suppose Node 1 is Rumor



$$P(\sigma_i|v_1) = P(\sigma_j|v_1) \text{ for all } \sigma_i, \sigma_j \in S(v, G_n)$$

Toy Example

Suppose Node 1 is Rumor



$$P(G_n|v^*) = \sum_{\sigma_i \in S(v^*, G_n)} P(\sigma_i|v^*)$$

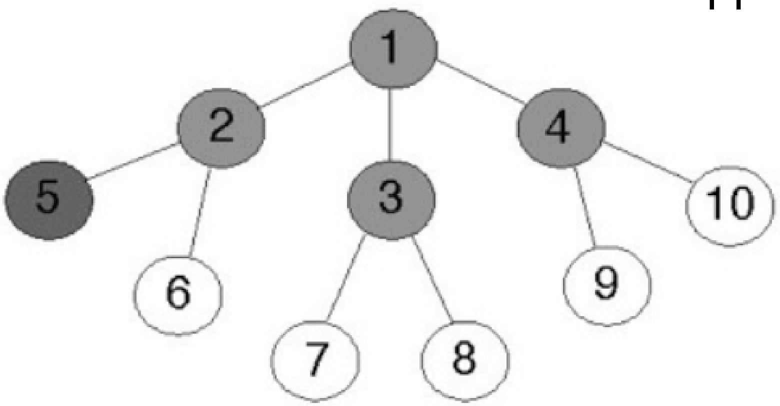
$$= |S(v^*, G_n)| \cdot P(\sigma|v^*) \qquad \forall \sigma_i \in S(v, G_n)$$

$$= |S(v^*, G_n)| \cdot \prod_{k=1}^{n-1} \frac{1}{dk - 2(k-1)}$$

$$\propto |S(v^*, G_n)|.$$

Toy Example

Suppose Node 1 is Rumor



compute each $P(G_4|v_i)$ by finding out the value $|S(v_i, G_n)|$.

they are of the same value $\frac{1}{3 \cdot 4 \cdot 5} = \frac{1}{60}$

$$P(G_n|v_1) = 3! \cdot \frac{1}{60}$$

$$P(G_n|v_2) = P(G_n|v_3) = P(G_n|v_4) = 1 \cdot 2! \cdot \frac{1}{60}$$

The Node with the Maximum Likelihood: v_1

Rumor Centrality

- **Rumor centrality counts the number of permitted permutation to spread a rumor**

$$R(v, G_n) = |S(v, G_n)|$$

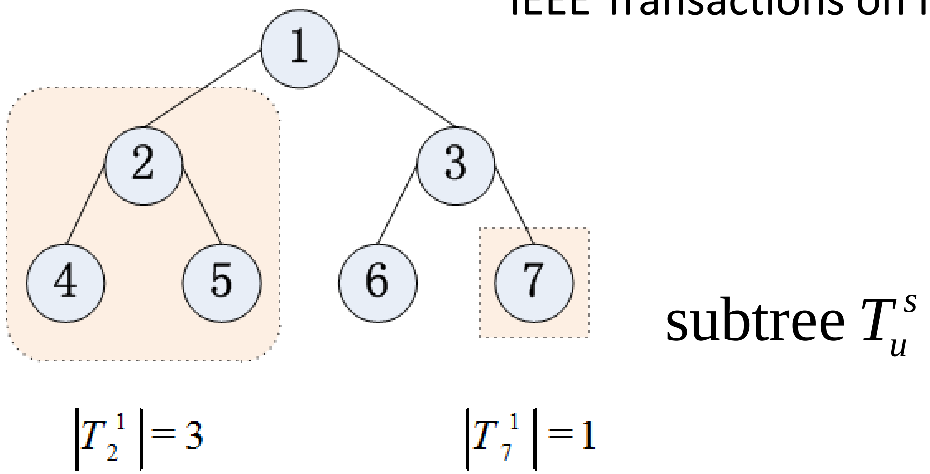
Shah and Zuman
IEEE Transactions on Information Theory 2011

Rumor Centrality

- **Rumor centrality counts the number of permitted permutation to spread a rumor**

$$R(s,G_n) = n!\prod_{u\in G_n} |T_u^s|^{-1}$$

Shah and Zuman
IEEE Transactions on Information Theory 2011



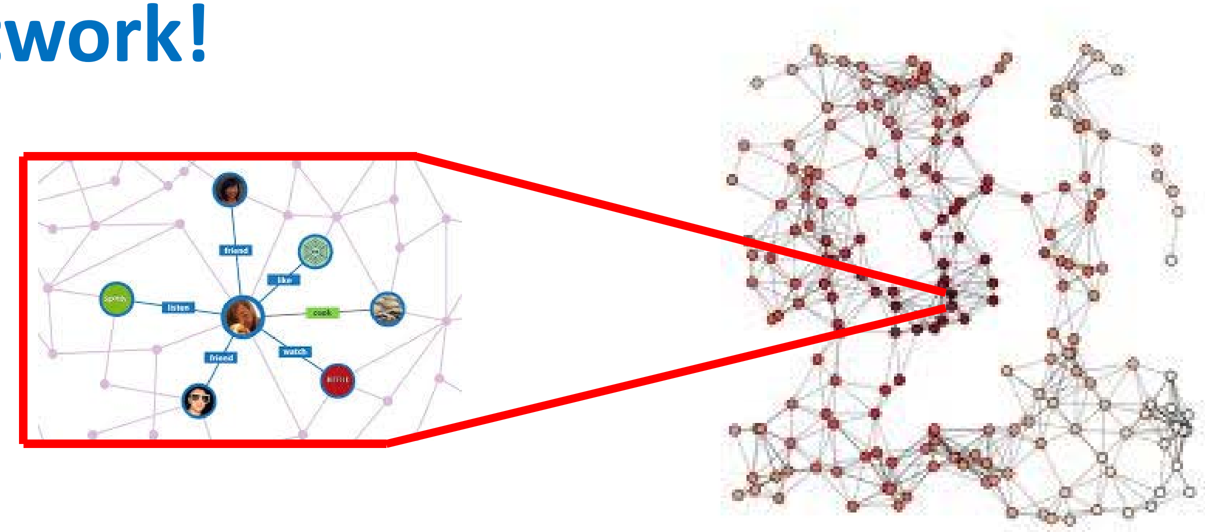
Rumor Center

- **ML (maximum likelihood) estimator** [Shah—TIT’11]

$$\widehat{v} = \arg\max_{v \in G_N} \mathbf{P}(G_N | v^* = v)$$

$$= \arg\max_{v \in G_N} R(v, G_N)$$

- **Most likely source is at the “center” of the network!**



Detection Probability $\mathbf{P_c}(n)$

- Detecting the most probable source by Bayes Theorem

Let G be a tree and G_n is a subtree of G at time n

v is the rumor center of G_n

Let the event $v = \mathit{source}$ denoted as S_v. $P(S_v|G_n)$ is the probability of the event that the rumor center is exactly the rumor source when given G_n

$$\mathbf{P_c}(n) = P(S_v|G_n) = \frac{P(G_n|S_v) \cdot P(S_v)}{\sum_{i \in G_n} [P(G_n|S_i) \cdot P(S_i)]}$$

Detection Probability

- Detecting the most probable source by Bayes Theorem

The probability of each vertex to be the source are equal, that is

$$P(S_i) = P(S_j) \ \ \forall\ i, j \in G_n,$$

and also we have $P(G_n|S_v) \propto R(v, G_n)$.

$$P(S_v|G_n) = \frac{P(G_n|S_v) \cdot P(S_v)}{\sum_{i \in G_n}[P(G_n|S_i) \cdot P(S_i)]}$$

=> $$P(S_v|G_n) = \frac{R(v, G_n)}{\sum_{i \in G_n} R(i, G_n)}$$

Detection Probability

- Bound

$$P(S_v|G_n) \leq \frac{1}{2}$$

no matter how large the size is or what shape of G_n is.

Shah and Zuman
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Detection Probability

- Bound

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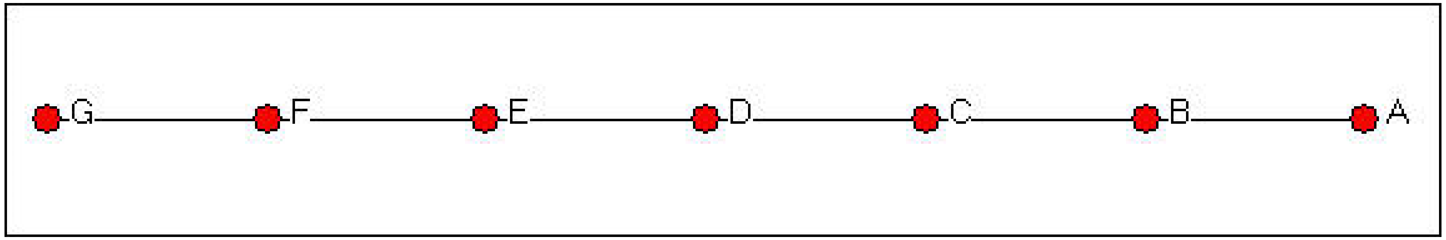
Minimum Detectability



- Line Network is undetectable!
- Can multiple observations help?

Minimum Detectability

- For finite *n*

$$
\begin{aligned}
\mathbf{P}_{\mathrm{c}}(n) &= \frac{1}{2} \sum_{\max\{x_1,x_2\}=n/2} \mathbf{P}_{\mathbf{G}}\left[\bigcap_{j=1}^{2}(X_j = x_j)\right] \\
&\quad + \sum_{\max\{x_1,x_2\}<n/2} \mathbf{P}_{\mathbf{G}}\left[\bigcap_{j=1}^{2}(X_j = x_j)\right], \\
&= \frac{1}{2^{n-1}}\binom{n-1}{\lfloor (n-1)/2 \rfloor}.
\end{aligned}
$$

Minimum Detectability

- For asymptotically large *n*

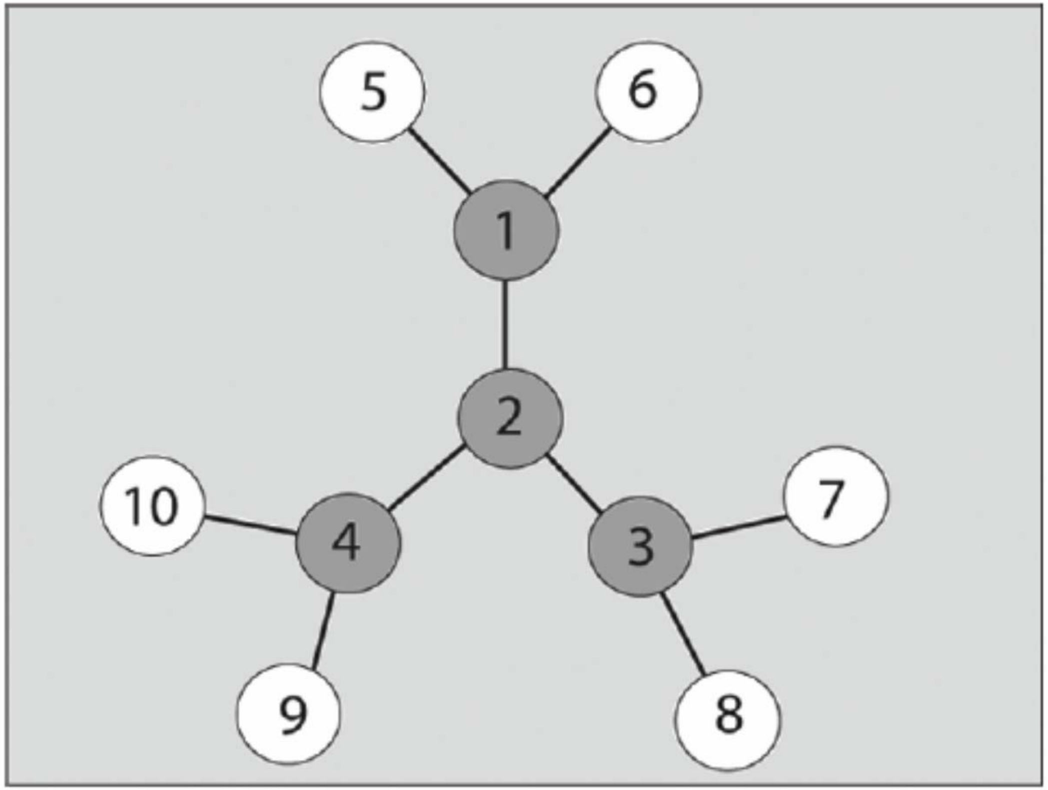
As $n \to \infty$, by the Stirling's formula, we have

$$
\begin{aligned}
\mathbf{P}_c(n) &\approx \frac{1}{2^n} \cdot \frac{n!}{[(n/2)!]^2} \\
&\approx \frac{1}{2^n} \cdot \frac{\sqrt{2\pi n} \cdot \left(\frac{n}{e}\right)^n}{\left[\sqrt{\pi n} \cdot \left(\frac{n}{2e}\right)^{n/2}\right]^2} \\
&= \sqrt{\frac{2}{\pi n}} \\
&= O\left(\frac{1}{\sqrt{n}}\right).
\end{aligned}
$$

Stirling's asymptotic formula:

$$n! \sim \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$$

Detectability



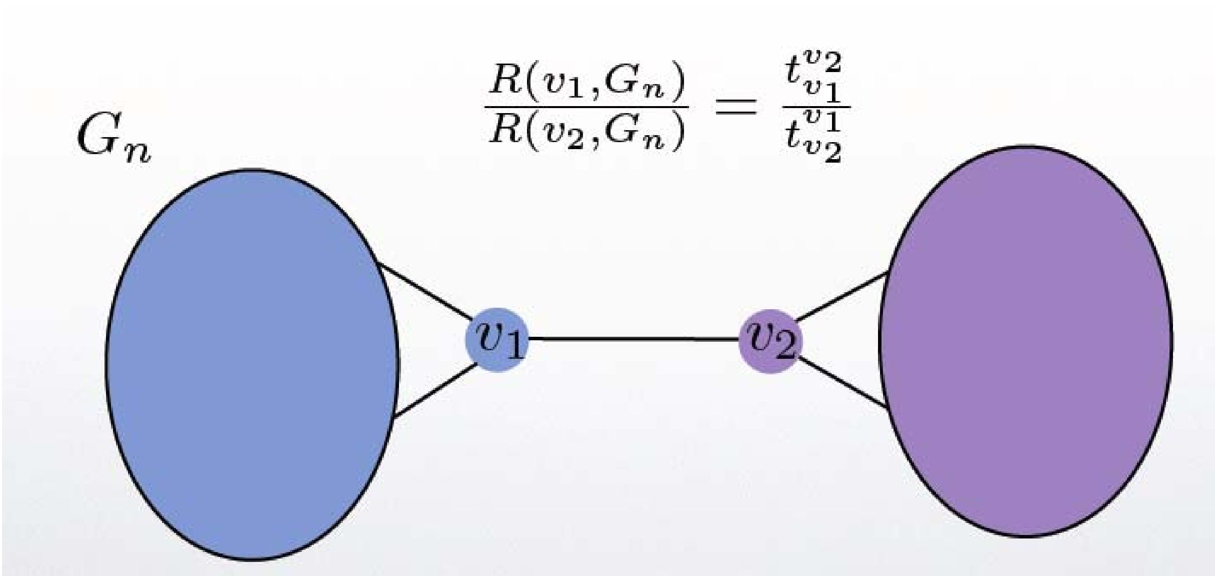
- Degree-regular tree with degree strictly larger than 2 is asymtotically detectable
- Phase-Transition like phenomenon!

Connection with Graph Convexity in Graph Theory

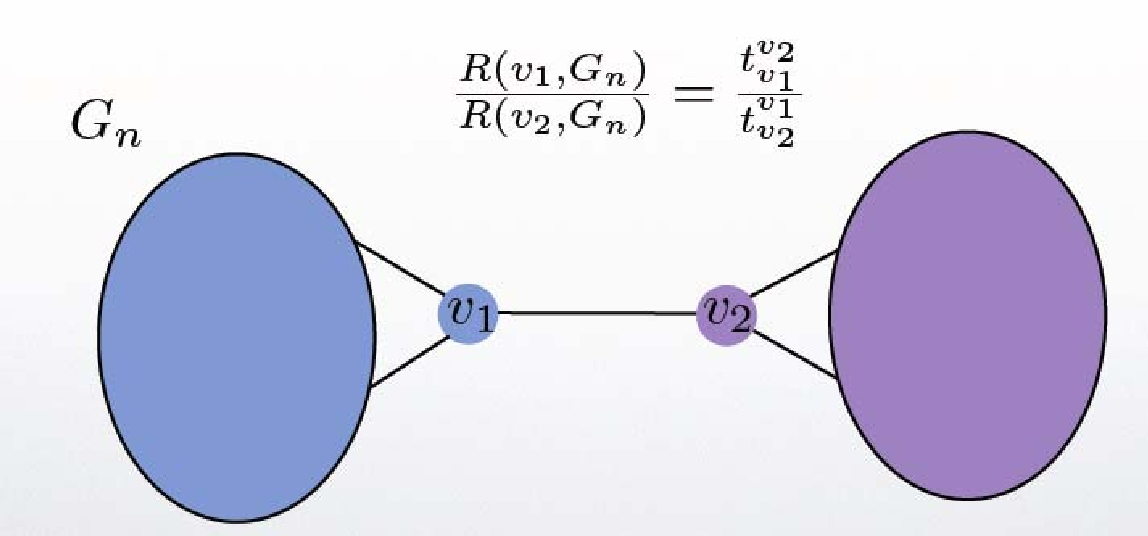
- Recall that $R(s, G_n) = n! \prod_{u \in G_n} |T_u^s|^{-1}$

Shah and Zuman
IEEE Transactions on Information Theory 2011

consider two adjacent vertices u,v in G_n and a vertex $w \in G_n - \{u, v\}$



Connection with Graph Convexity in Graph Theory



- Since $t_u^v = n - t_v^u$ and $t_w^v = t_w^u$

$$\frac{P(u|G_n)}{P(v|G_n)} = \frac{R(u,G_n)}{R(v,G_n)} = \frac{t_u^v}{n - t_v^u}$$

Alternative Characterization of Rumor Cente

Given an n vertices tree G_n. $v \in G_n$ is a rumor center

if and only if

$$t_u^v \leq \frac{n}{2}$$

for all $u \in G_n - \{v\}$

Shah and Zuman
IEEE Transactions on Information Theory 2011

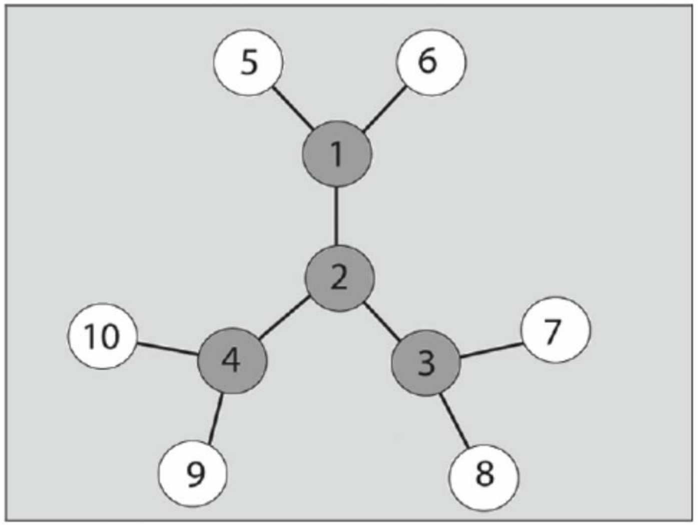
Connection with Graph Theory

- Distance Centrality (Social Network Analysis)

$$D(v, G_n) = \sum_{j \in G_n} d(v, j)$$

where $d(v, j)$ is the shortest path length between *v* and *j (eccentricity)*

- Vertex in G_n with minimum distance centrality is called *distance center*



Distance from 1 to all shaded nodes:
From Node 1 to Node 2: 1
From Node 1 to Node 3: 2
From Node 1 to Node 4: 2
$\Rightarrow$Distance Centrality of Node 1 = 1+2+2=5

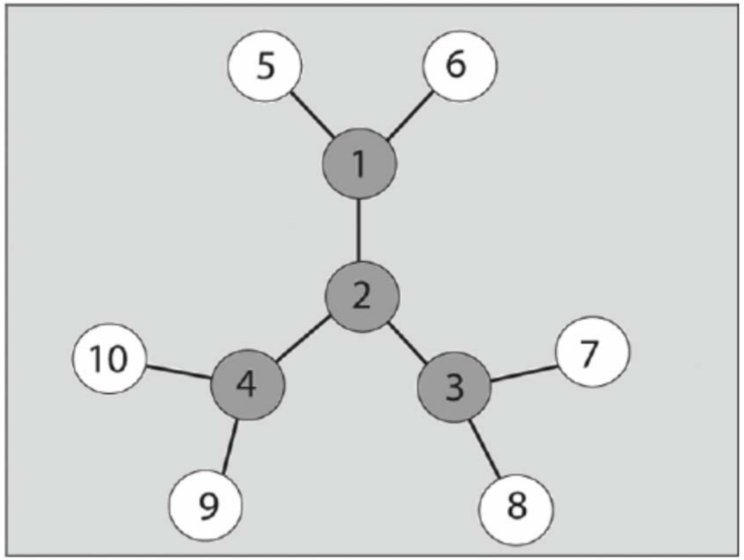
Distance Centrality of Node 2 is 3, therefore Node 2 is *Distance Center*

Connection with Graph Theory

- Mass of graph (Graph Theory)

$$weight(v) = \ max_{u \in child(v)} \{t_c^v\}$$

- Vertex in G_n with minimum weight is called *mass center* Bohdan Zelinka,“Medians and Peripherians of Trees’, Archivum Mathematicum, Vol. 4 (1968), No.2, 87–95.



Subtree of a child (Node 2) of Node 1 has size of 3
⇒Weight of Node 1 = 3

Weight of Node 2 is 1 for its three subtrees, therefore Node 2 is *Mass Center*

Connection with Graph Theory

Let G_n be an infected subtree of G and v be a vertex in G. Then the following statements are equivalent:

1. v is a *distance center* of G_n.
2. v is a *rumor center* of G_n.
3. v is a *mass center* of G_n.

Connection with Graph Theory

Let G_n be an infected subtree of G and v be a vertex in G. Then the following statements are equivalent:

1. v is a *distance center* of G_n.
2. v is a *rumor center* of G_n.
3. v is a *mass center* of G_n.

- A tree has either exactly one or two mass centers (graph theory result)
- There are at most two rumor centers when the maximum subtree tree is *n/2* (Shah's IT result)

Algorithms

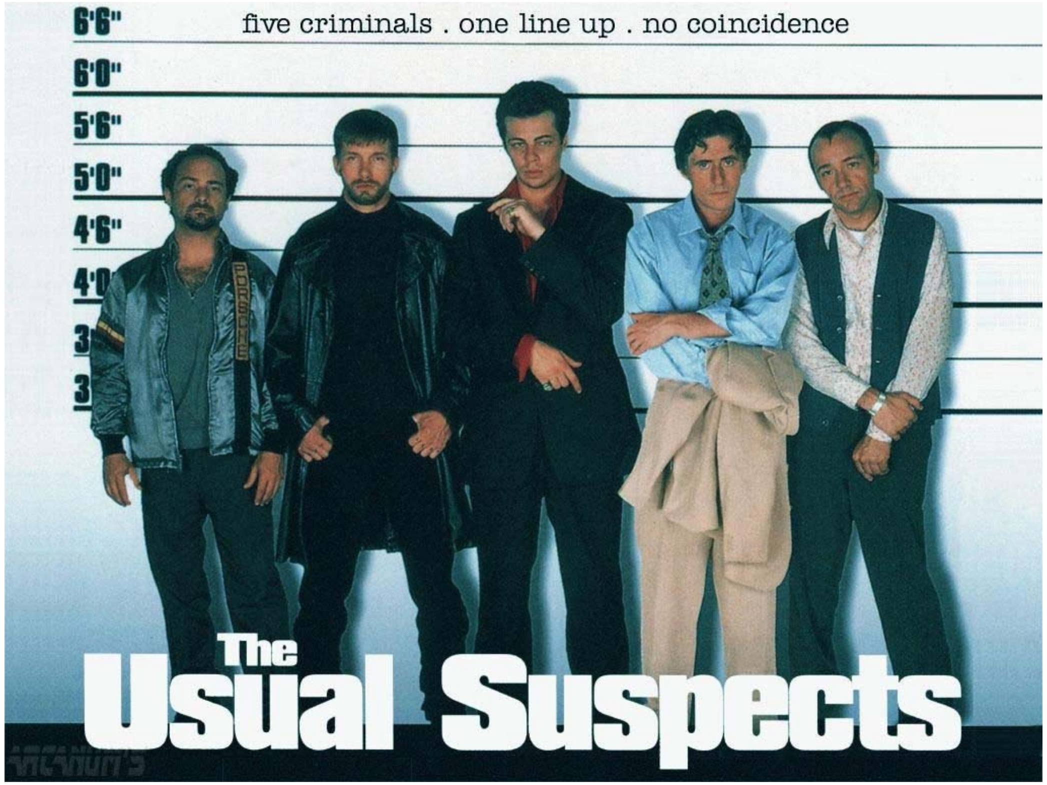
- A myriad of polynomial-time algorithms for computing rumor center
  - Mass center algorithm (graph theory)
  - Distance center algorithm (social network analysis)
  - Message passing algorithm (Shah's IT result)
  - …etc

Detection with Suspects

- **Why consider suspects?**
  - Not all infected are suspects
  - spread of infectious disease from cities to cities

    (frequent travellers)
  - infection of rumors or computer viruses in cyberspace

    (vulnerable hosts)



– **Suspect characteristics significantly affect detectability and add an interesting dimension to identifying the source reliably.**

Detection with Suspects

- **What's new with suspects?**

Finite regime:

– at most vs. at least 0.5 detection probability

Asymptotic regime:

– 0.307 vs. 1 best detection probability

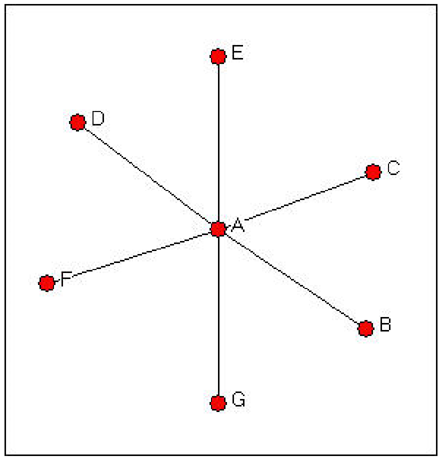
Insightful monotonicity and averaging results

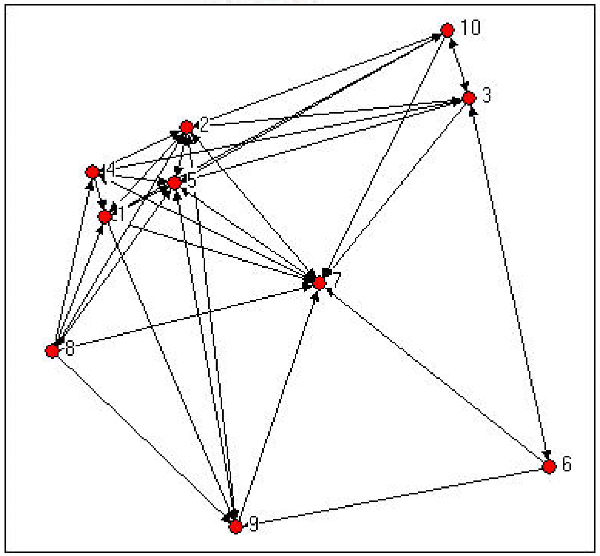
Dong, Zhang and Tan
Proc. of IEEE Symposium on Information Theory 2013

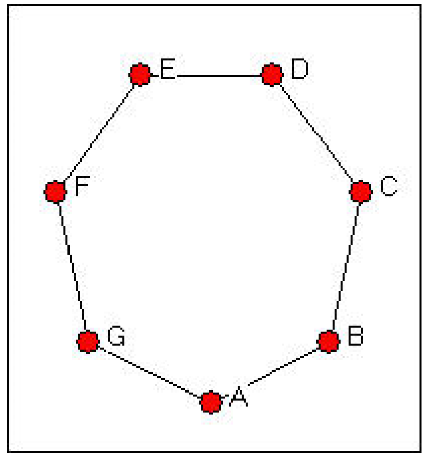
Zhao, Dong, Zhang and Tan
ACM SIGMETRICS 2014

Detectability







– Detectability is graph constrained. Network connectivity matters!

MAP Rumor Source Estimator

- **MAP (maximum a posteriori) estimator**
- A prior suspect set S
- An observation of n infected nodes G_n

$$\hat{s} = \arg\max_{s \in \{S \bigcap G_n\}} \mathrm{P}_G\{s \mid G_n\} = \arg\max_{s \in \{S \bigcap G_n\}} \mathrm{P}_G\{G_n \mid s\}$$

MAP Rumor Source Estimator

- **For regular tree-type networks**

$$\hat{s} = \arg\max_{s \in \{S \cap G_n\}} R(s, G_n)$$

- optimal MAP estimator
- focus on regular trees

- **For general tree-type networks**

$$\hat{s} = \arg\max_{s \in \{S \cap G_n\}} R(s, G_n)$$

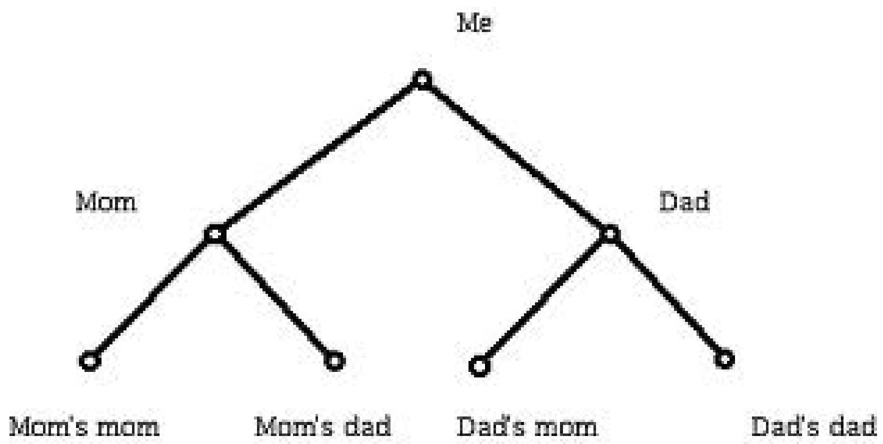
- Analyze the correct detection probability upon observing *n* infected nodes

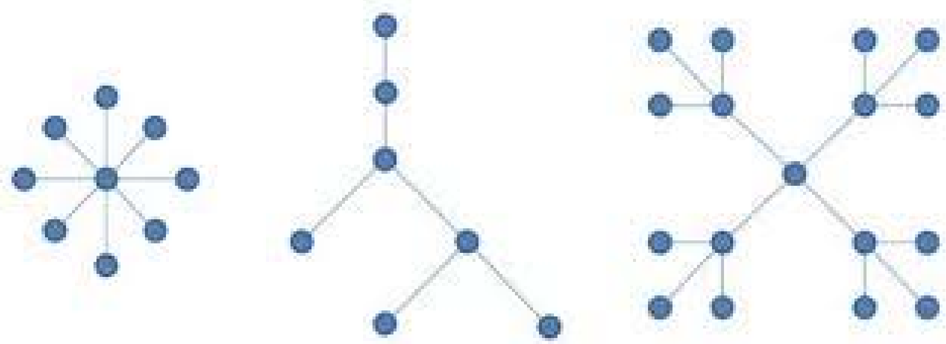
$$\mathrm{P}_c(n) = \mathrm{Prob}\left[\hat{s} = s^*\right]$$

- **For general networks**

$$\hat{s} = \arg\max_{s \in \{S \cap G_n\}} R\left(s, T_{\mathrm{bfs}}(s)\right)$$

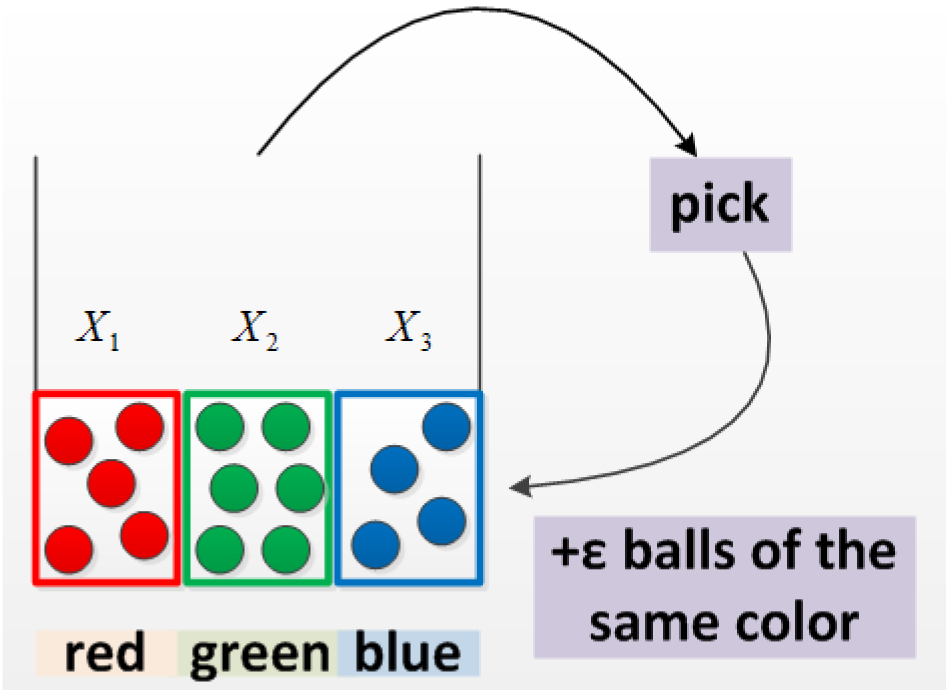
Detectability on Tree





- How to detect on a tree?
- Can fewer number of infected nodes help detectability?
- Can a higher degree help detectability?

Pólya's Urn Model



[Johnson—JWS'97]

- Joint distribution

$$\mathrm{P}_G\left[\bigcap_{j=1}^{\delta}\left(X_j = x_j\right)\right]$$

- Marginal distribution

$$\mathrm{P}_G\left[X_1 = x_1\right]$$

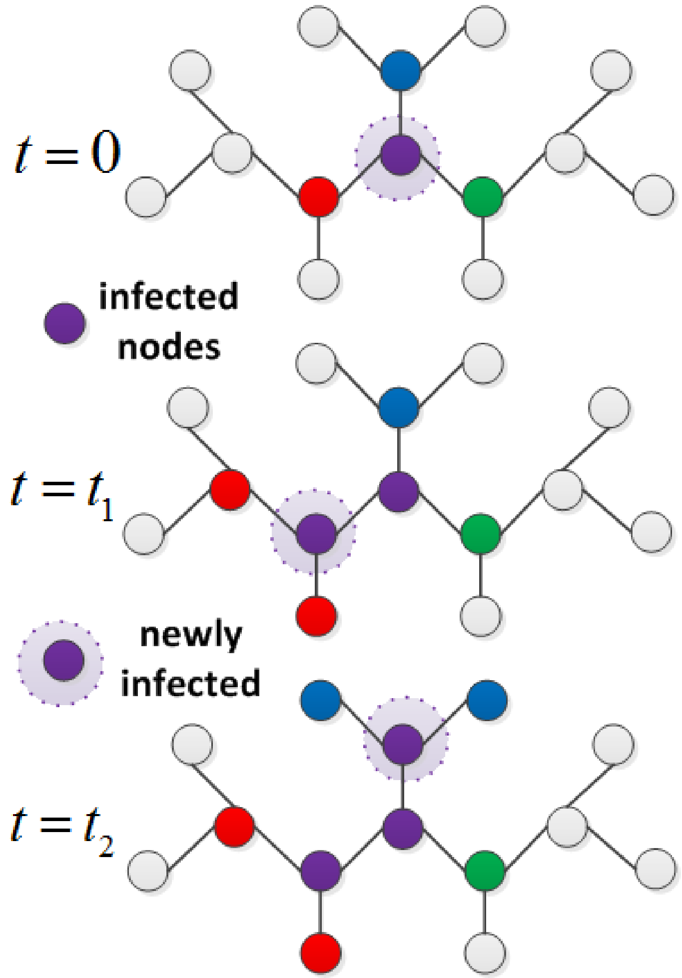
- Limit distributions

$$\lim_{n\to\infty}\mathrm{P}_G\left[\bigcap_{j=1}^{\delta}\left(\frac{X_j}{n} = y_j\right)\right]$$

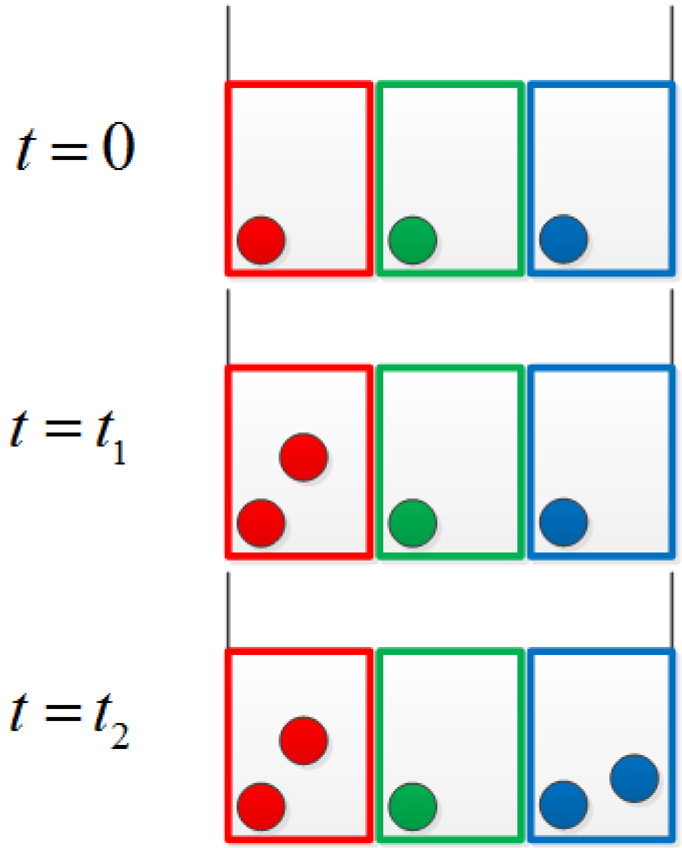
$$\lim_{n\to\infty}\mathrm{P}_G\left[\frac{X_1}{n} = y_1\right]$$

Equivalence to Pólya’s Urn Model

- **Rumor spreading process**



- **Ball drawing process**



$$s = \delta - 2$$

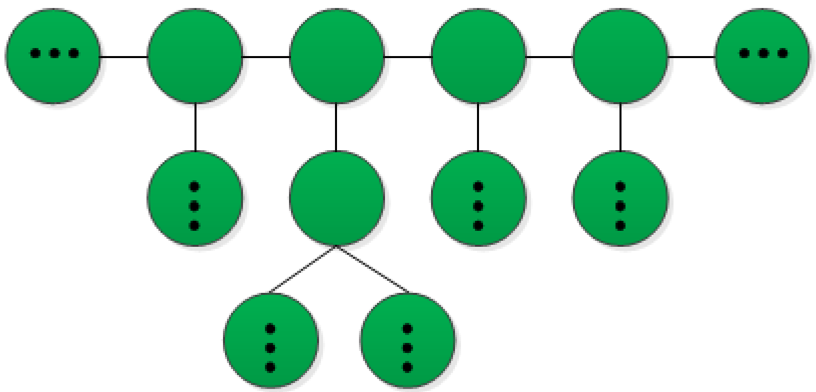
Suspecting all Nodes

----main results

Case 1

– Any infected node might be the rumor source.

[Shah—TIT'11, Shah—SIGMETRICS'12]



– **The detection probability is asymptotically upper bounded by 0.307.**

Main results

- node degree $\delta = 2$

$$P_c(n) = \frac{1}{2^{n-1}} \binom{n-1}{\lfloor (n-1)/2 \rfloor} \sim O(1/\sqrt{n})$$

- node degree $\delta = 3$

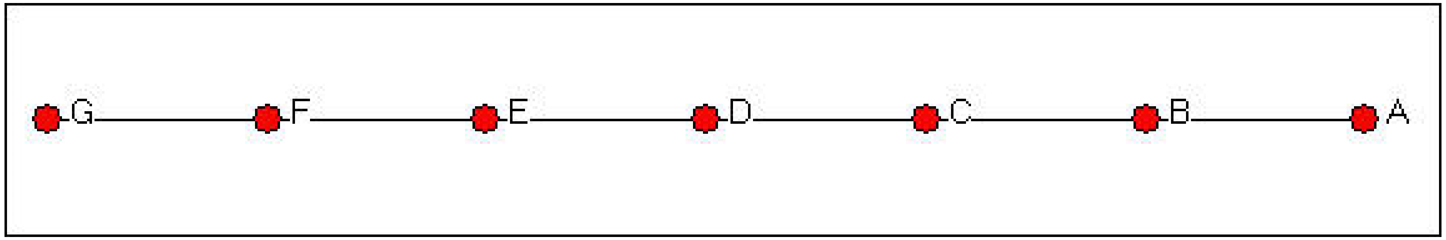
$$P_c(n) = \frac{1}{4} + \frac{3}{4} \frac{1}{2\lfloor n/2 \rfloor + 1} \sim \frac{1}{4} + O(1/n)$$

- node degree $\delta > 3$

$$\lim_{n\to\infty} P_c(n) = 1 - \delta \left(1 - I_{1/2}\left(\frac{1}{\delta - 2}, \frac{\delta - 1}{\delta - 2} \right) \right) \to 0.307$$

- Monotonicity: Detection probability increases with degree and decreases with *n*

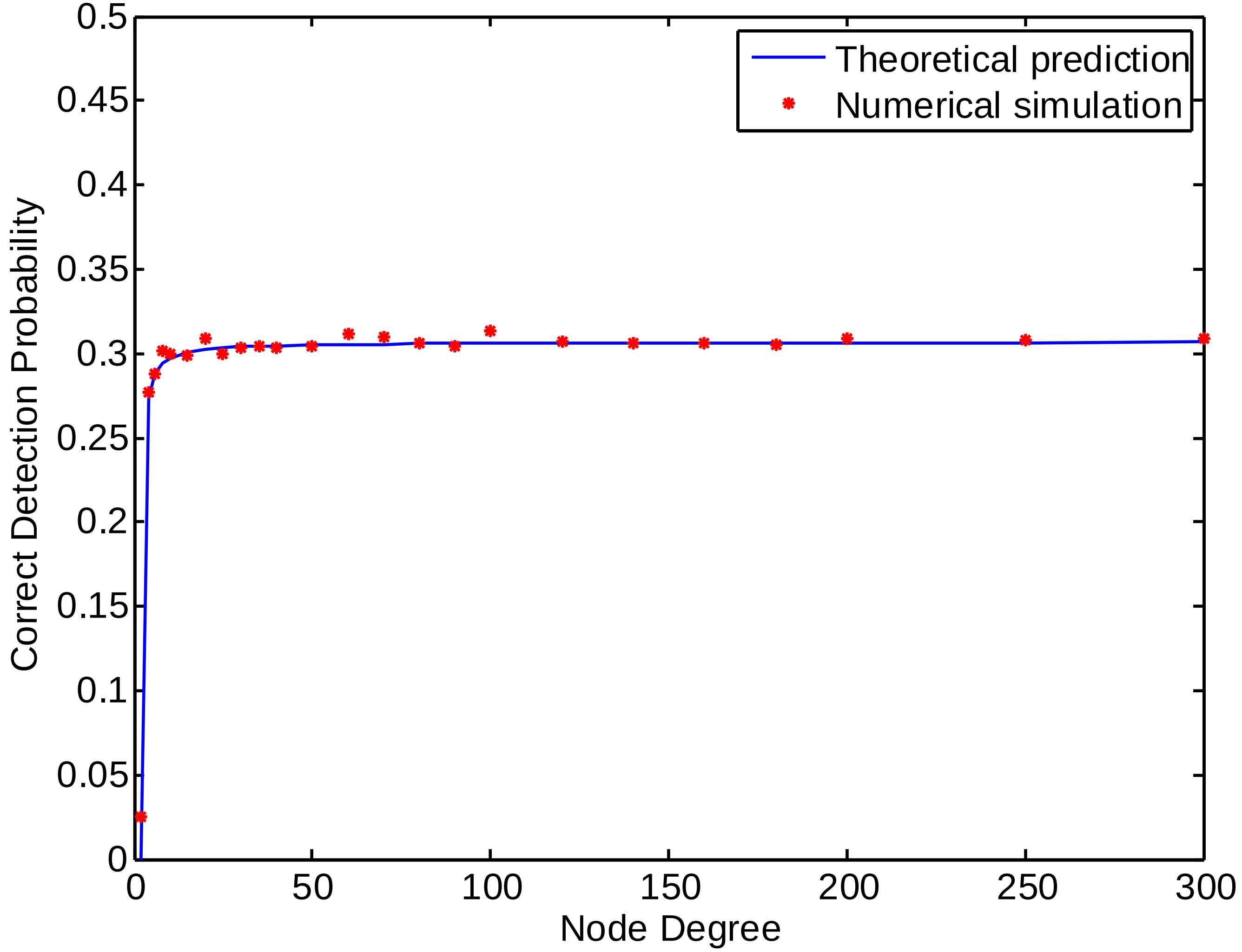
Minimum Detectability



- Line Network is undetectable!
- Can multiple observations help?

Suspecting all Nodes

----validation experiment

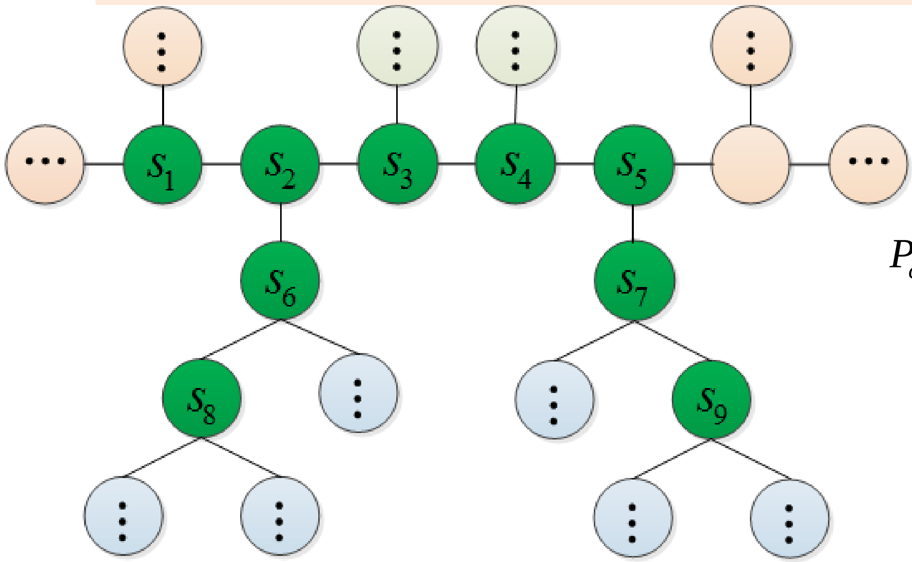


Connected Suspects

----main results

Case 2

– All suspect nodes form a connected subgraph.



– **The performance is significantly improved and reliable detection can be achieved.**

Main results

- node degree $\delta = 2$

$$P_c(n) = \frac{1}{k}\left(1 + \frac{k-1}{2^{n-1}}\binom{n-1}{\lfloor (n-1)/2 \rfloor}\right) \sim \frac{1}{k} + \mathrm{O}(1/\sqrt{n})$$

- node degree $\delta = 3$

$$P_c(n) = \frac{k+1}{2k} + \frac{k-1}{k}\frac{1}{4\lfloor n/2 \rfloor + 2} \sim \frac{k+1}{2k} + \mathrm{O}(1/n) \geq \frac{1}{k} + \frac{k-1}{2k}$$

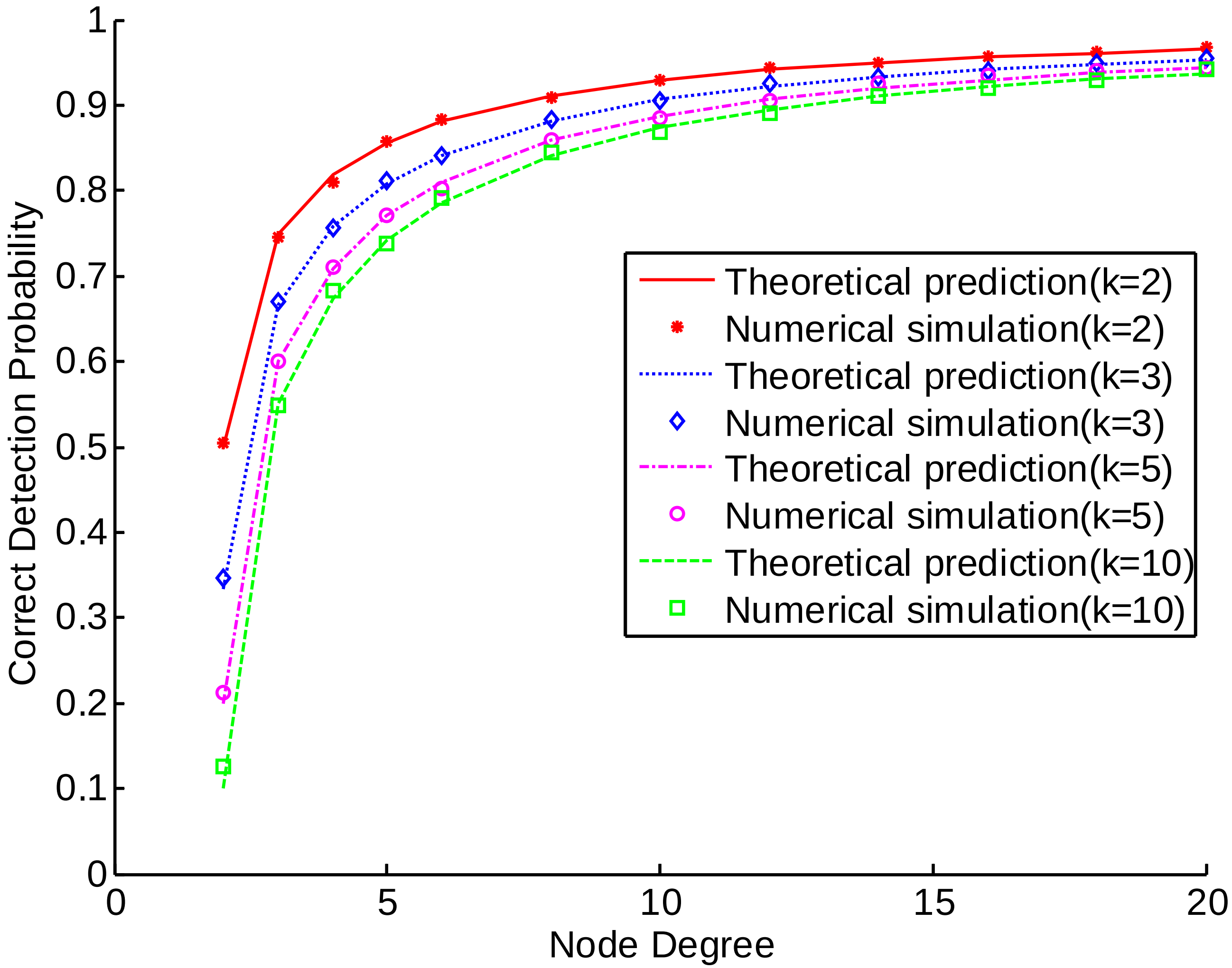
- node degree $\delta > 3$

$$\lim_{n\to\infty} P_c(n) = 1 - \frac{2k-2}{k}\left(1 - I_{1/2}\left(\frac{1}{\delta-2}, \frac{\delta-1}{\delta-2}\right)\right) > \frac{1}{k} + \frac{k-1}{2k} \to 1$$

- Monotonicity: Detection probability increases with degree and decreases with *n*

Connected Suspects

----validation experiment



Connected Suspects

----with vs. without prior knowledge

Closer-up look at case 2

- node degree $\delta > 2$

$$\lim_{n\to\infty} P_c(n) = 1 - \frac{2k-2}{k}\left(1 - I_{1/2}\left(\frac{1}{\delta-2}, \frac{\delta-1}{\delta-2}\right)\right)$$

- exceed prior probability

$$P_c(n) \geq \frac{1}{k} + \frac{k-1}{2k}$$

- at least 0.5-detection

$$P_c(n) \geq 2I_{1/2}\left(\frac{1}{\delta-2}, \frac{\delta-1}{\delta-2}\right) - 1 \geq 0.5$$

- achieve reliable detection

$$\lim_{\delta\to\infty}\lim_{n\to\infty} P_c(n) = 1$$

Comparison with case 1

- node degree $\delta > 2$

$$\lim_{n\to\infty} \mathrm{P}_c(n) = 1 - \delta\left(1 - I_{1/2}\left(\frac{1}{\delta-2}, \frac{\delta-1}{\delta-2}\right)\right)$$

- non-trivial positive value

$$P_c(n) > 0$$

- at most 0.5-detection

$$P_c(n) \leq 0.5$$

- upper-bounded by 0.307

$$\lim_{\delta\to\infty}\lim_{n\to\infty} P_c(n) = 0.307$$

– **Suspect characteristics (connectivity) bring about new ingredients.**

Connected Suspects

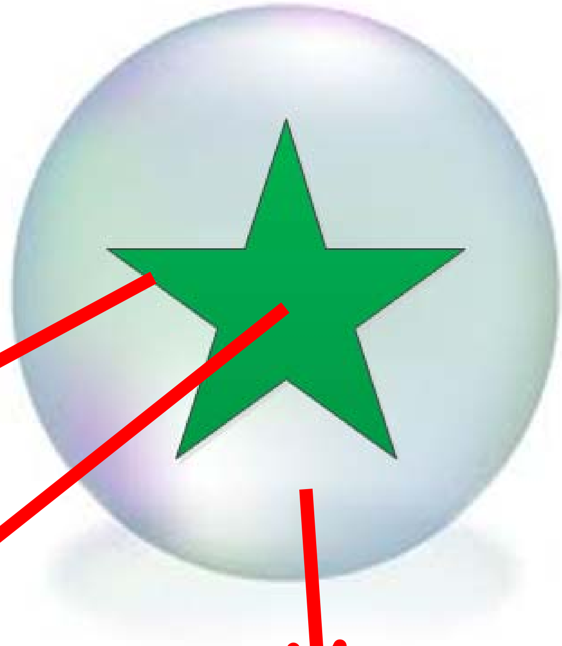
----no degeneration with large *k*

Averaged by Bayes' Rule

$$
\begin{aligned}
\mathrm{P}_c(n) &= \sum_{i=1}^{k} \mathrm{P}_s(s_i)\,\mathrm{P}_c(n|s_i) \\
&= \frac{1}{k}\sum_{s^*\in S} \mathrm{P}_c(n|s^*)
\end{aligned}
$$

- Suspect nodes nearby the subgraph boundary are easier to identify.
- Suspect nodes inside the connected subgraph are harder to identify.

Explanation

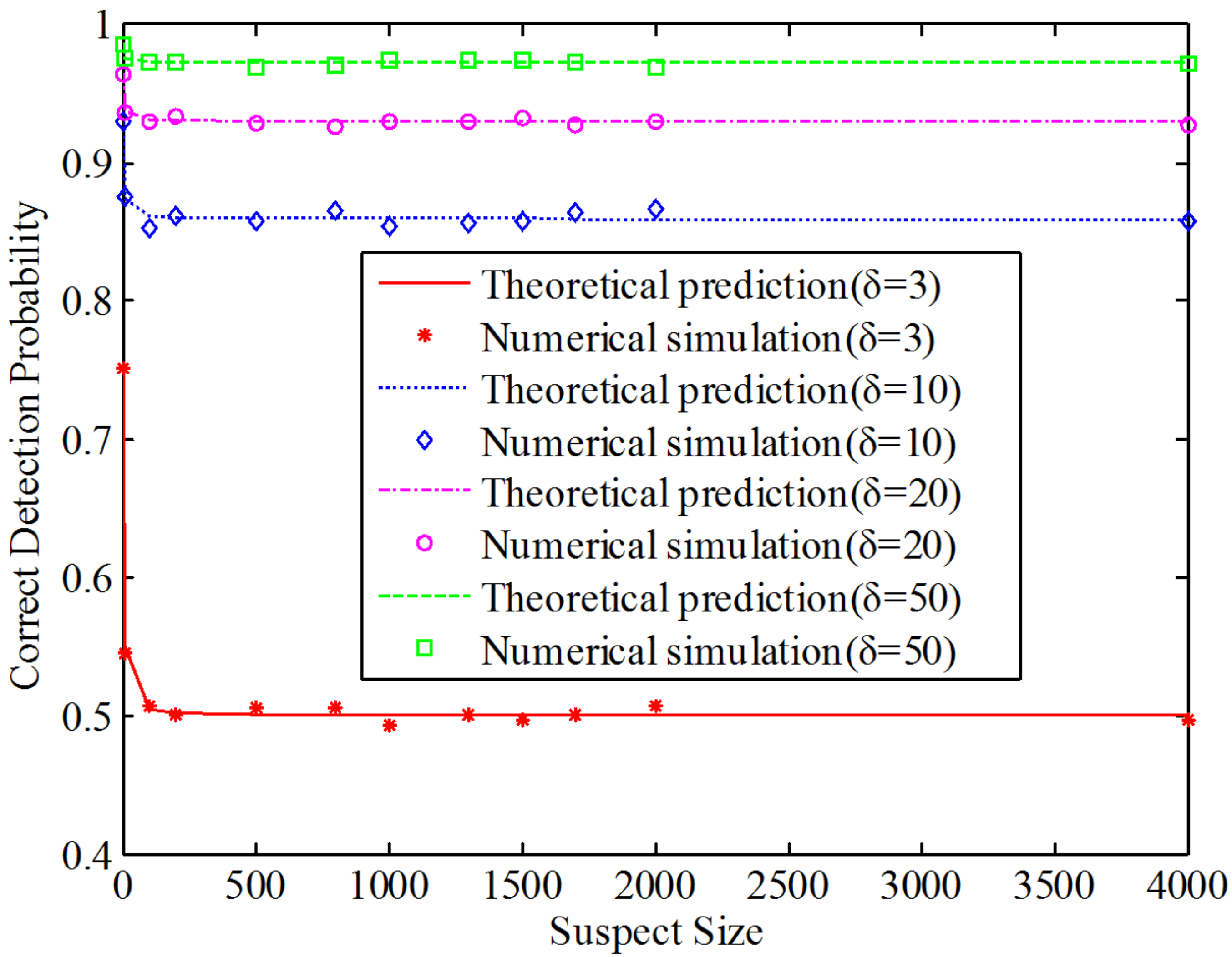


- infinite regular-tree network

- **The results in case 2 don't degenerate to case 1 with large *k*.**

Connected Suspects

----validation experiment

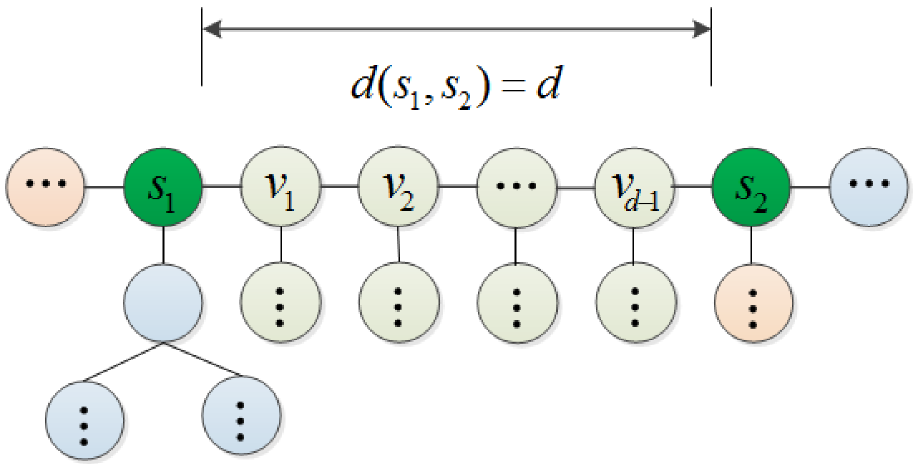


Two Suspects

----main results

Case 3

– Two suspect nodes is separated by *d.*



– **Identifying the rumor source is more difficult if the two suspects are closer.**

Main results

- node degree $\delta = 2$

$$\mathrm{P_c}(n) = \begin{cases} \frac{1}{2} - \sum_{z=(n-d-1)/2}^{(n+d+1)/2} \binom{n-1}{z}, (n-d) \text{ is odd} \\ \frac{1}{2} - \sum_{z=(n-d)/2}^{(n+d-2)/2} \binom{n-1}{z}, (n-d) \text{ is even} \end{cases}$$

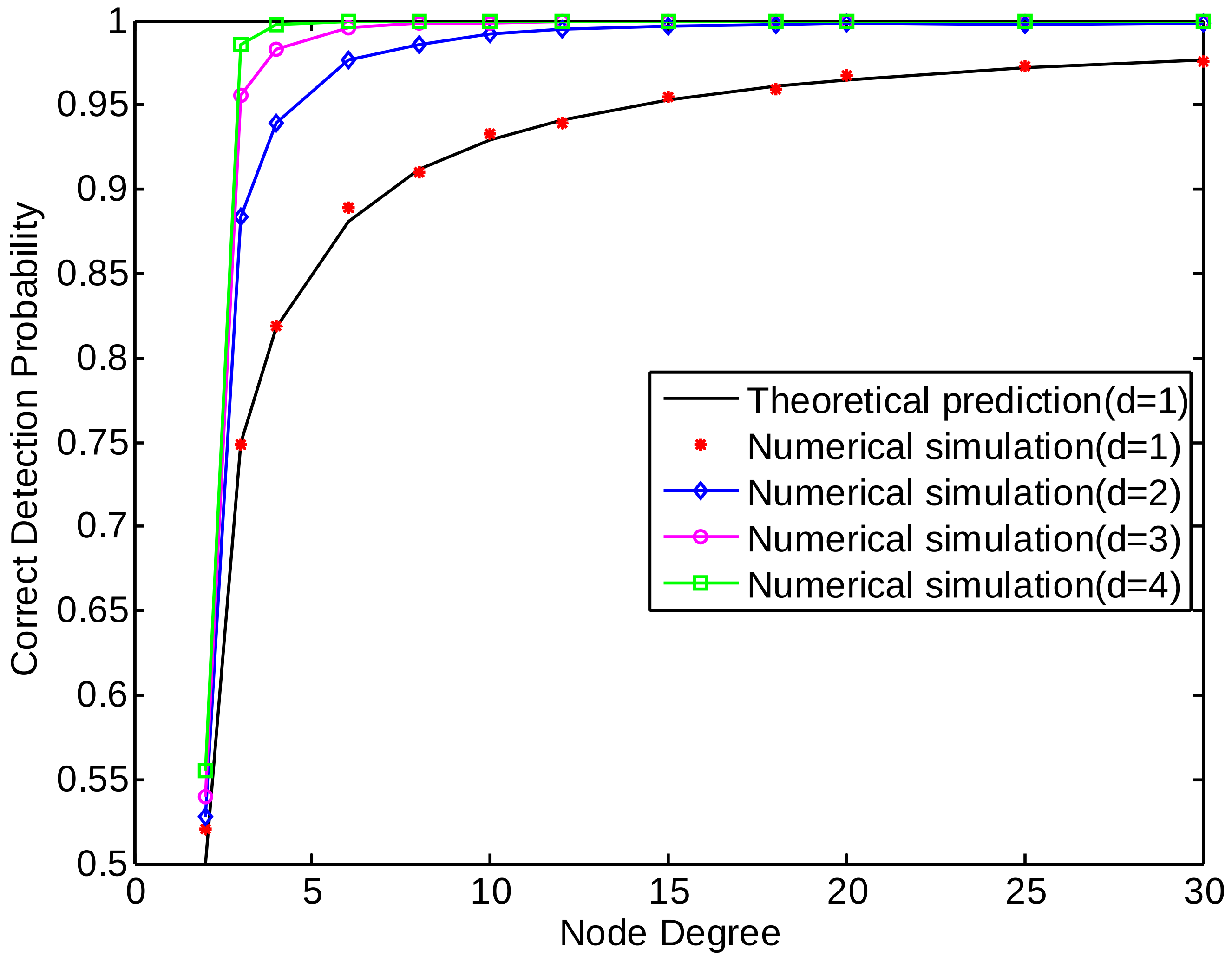
- node degree $\delta > 2$

$$\lim_{n\to\infty} P_c(n) = I_{1/2}\left(\frac{1}{\delta-2}, \frac{\delta-1}{\delta-2}\right) \geq 0.75, d = 1 \to 1$$

- Monotonicity: Detection probability increases with *d*

Two Suspects

----validation experiment



SIR Spreading Model

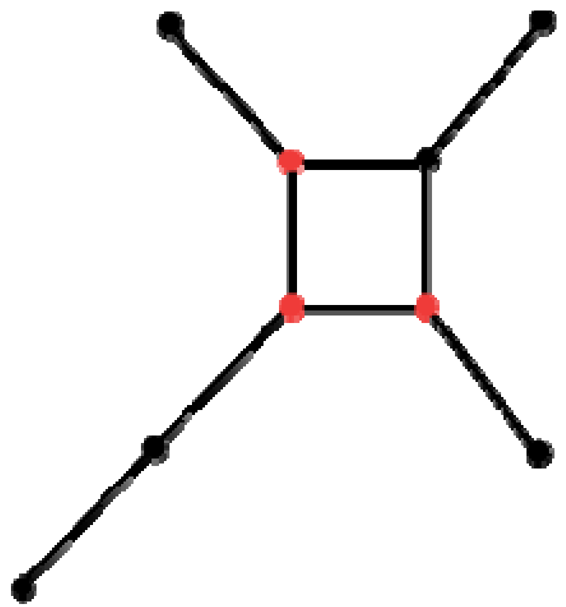
- **Fixed Population *N (only one infected at each* t*)***
- **Susceptible Set at time *t*** $S(t)$
- **Infected Set at time *t*** $I(t)$
- **Recovery Set at time *t***
- **Different parameter configuration with infectious rate and recovery rate**

N. T. J. Bailey, *The Mathematical Theory of Infectious Diseases and its Applications*, second edition, Griffin, London, 1975.

Maximum Likelihood Estimator in SIR Spreading

- **Sample-path analysis**
- **Jordan center (graph theory) of a graph is the set of all vertices of minimum eccentricity**

$$\gamma_s \in \arg\min_{b \in \mathcal{V}} \left(\max_{a \in \mathcal{V}_I^{(s)}} d(b,a) \right)$$



- **Used for single-source and multiple-source detection** [Luo—TSP'13, Zhu—ITA'13]

Detection for General Tree

- Still an open problem
  - Each permitted permutation does not have equal probability

$$P(\sigma_i|v_1) = \prod_{k=1}^{n-1} \frac{1}{\sum_{v_i \in V(G_k)} d(v_i) - 2(k-1)}$$

- Breadth-First-Search (BFS) Heuristic Algorithm
  - Rumor centrality algorithm on BFS tree

Detection for General Graph

- Still an open problem
  - How to deal with loops?

- Breadth-First-Search (BFS) Heuristic Algorithm
  - BFS tree approximates *diffusion tree*
  - Rumor centrality algorithm on BFS tree

Detection for General Graph

- Maximum Likelihood Estimator:

$$\hat{s} \in \arg\max_{s \in G_N^K} P(G_N^K \mid s)$$

- where $P(G_N^K \mid s) = \sum_{m=1}^{M} P(\sigma_m \mid s)$

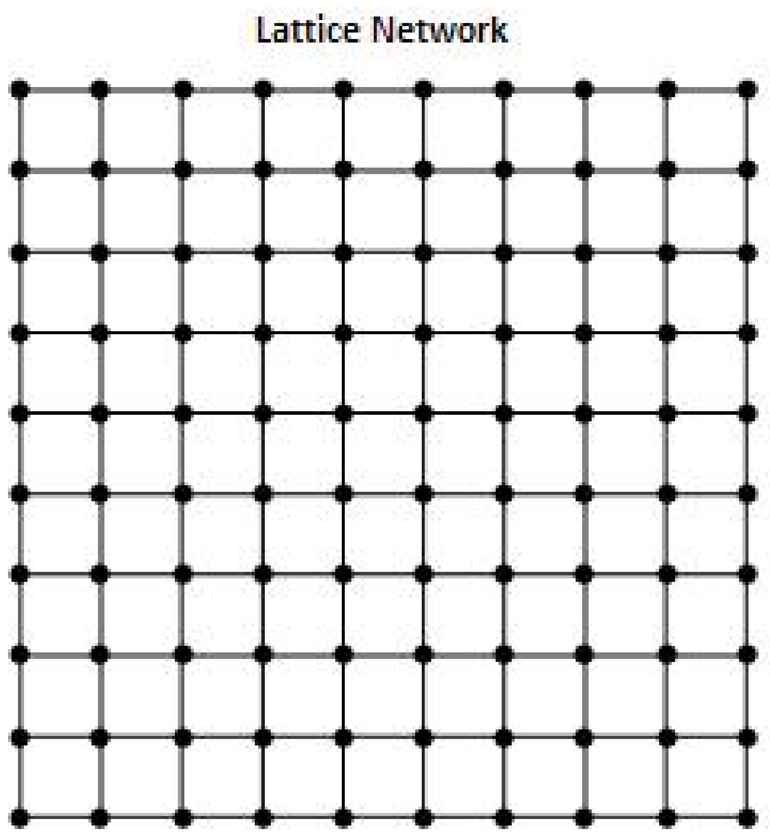
$\mathcal{I}_k(\sigma \mid s)$ is the number of infected neighboring nodes of the kth node in $\sigma(s)$ at time $k-1$

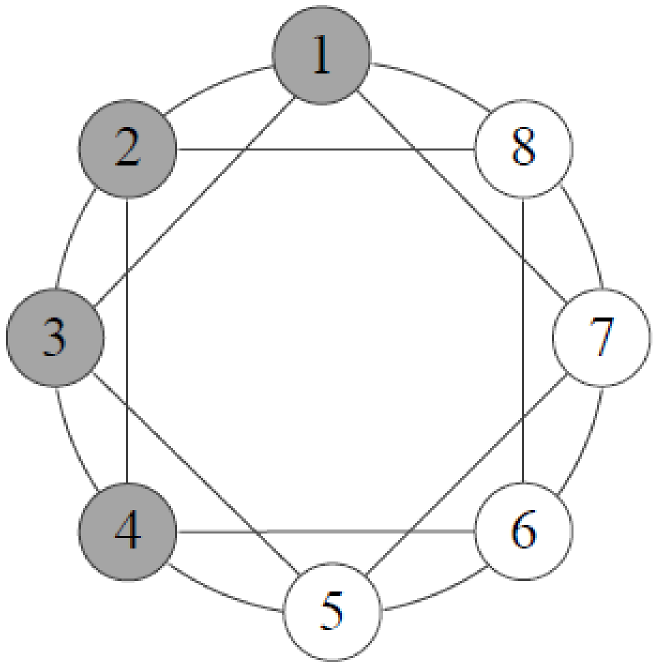
$\mathcal{S}_k(\sigma \mid s)$ is the total number of uninfected neighboring nodes for all the infected nodes in $G_N^k(\sigma \mid s)$.

$$P(\sigma_m \mid s) = \prod_{k=2}^{K} \frac{\mathcal{I}_k(\sigma_m \mid s)}{\mathcal{S}_{k-1}(\sigma_m \mid s)} \qquad P(G_N^K \mid s) = \sum_{m=1}^{M} \left(\prod_{k=2}^{K} \frac{\mathcal{I}_k(\sigma_m \mid s)}{\mathcal{S}_{k-1}(\sigma_m \mid s)} \right)$$

Detection for General Graph

- Toy Example for Regular Lattice Graph



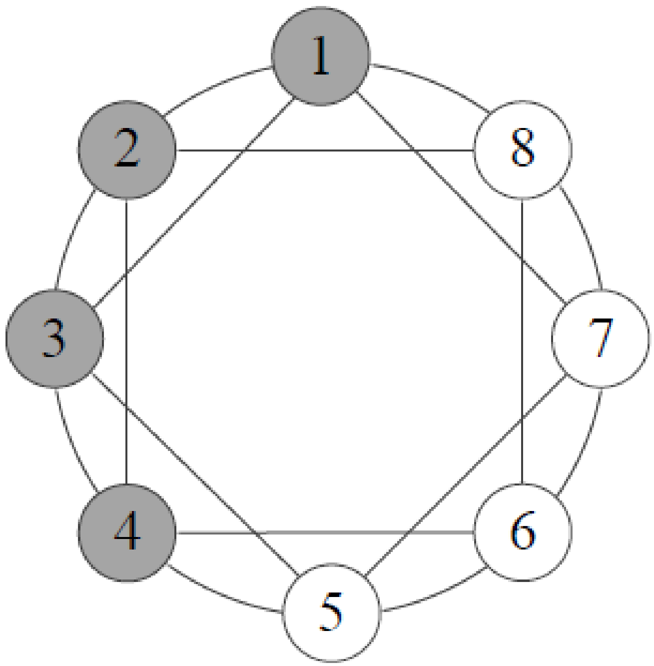


Detection for General Graph

- Toy Example for Regular Lattice Graph

$$\sigma_1 : s_1 \to s_2 \to s_3 \to s_4, \quad \sigma_2 : s_1 \to s_2 \to s_4 \to s_3,$$

$$\sigma_3 : s_1 \to s_3 \to s_2 \to s_4, \quad \sigma_4 : s_1 \to s_3 \to s_4 \to s_2.$$



$$P(\sigma_1 \,|\, s_1) = \frac{1}{4} \cdot \frac{2}{6} \cdot \frac{2}{6} = \frac{1}{36}, \quad P(\sigma_2 \,|\, s_1) = \frac{1}{4} \cdot \frac{1}{6} \cdot \frac{3}{8} = \frac{3}{192},$$

$$P(\sigma_3 \,|\, s_1) = \frac{1}{4} \cdot \frac{2}{6} \cdot \frac{2}{6} = \frac{1}{36}, \quad P(\sigma_4 \,|\, s_1) = \frac{1}{4} \cdot \frac{1}{6} \cdot \frac{3}{8} = \frac{3}{192}.$$

exact likelihood of Node 1 is $P(G_8^4 \,|\, s_1) = P(\sigma_1 \,|\, s_1) + P(\sigma_2 \,|\, s_1) + P(\sigma_3 \,|\, s_1) + P(\sigma_4 \,|\, s_1) = \frac{25}{288}$.

Detection for General Graph

- Toy Example for Regular Lattice Graph

For a d-regular lattice network, every nodes have the same degree d so we can replace $\mathcal{S}_{k-1}(\sigma \mid s)$ by $d(k-1) - d(G_N^{k-1}(\sigma))$

where $d(G_N^{k-1}(\sigma))$ is the total number of degree for the subgraph $G_N^{k-1}(\sigma)$ with the first $k-1$ nodes in σ infected.

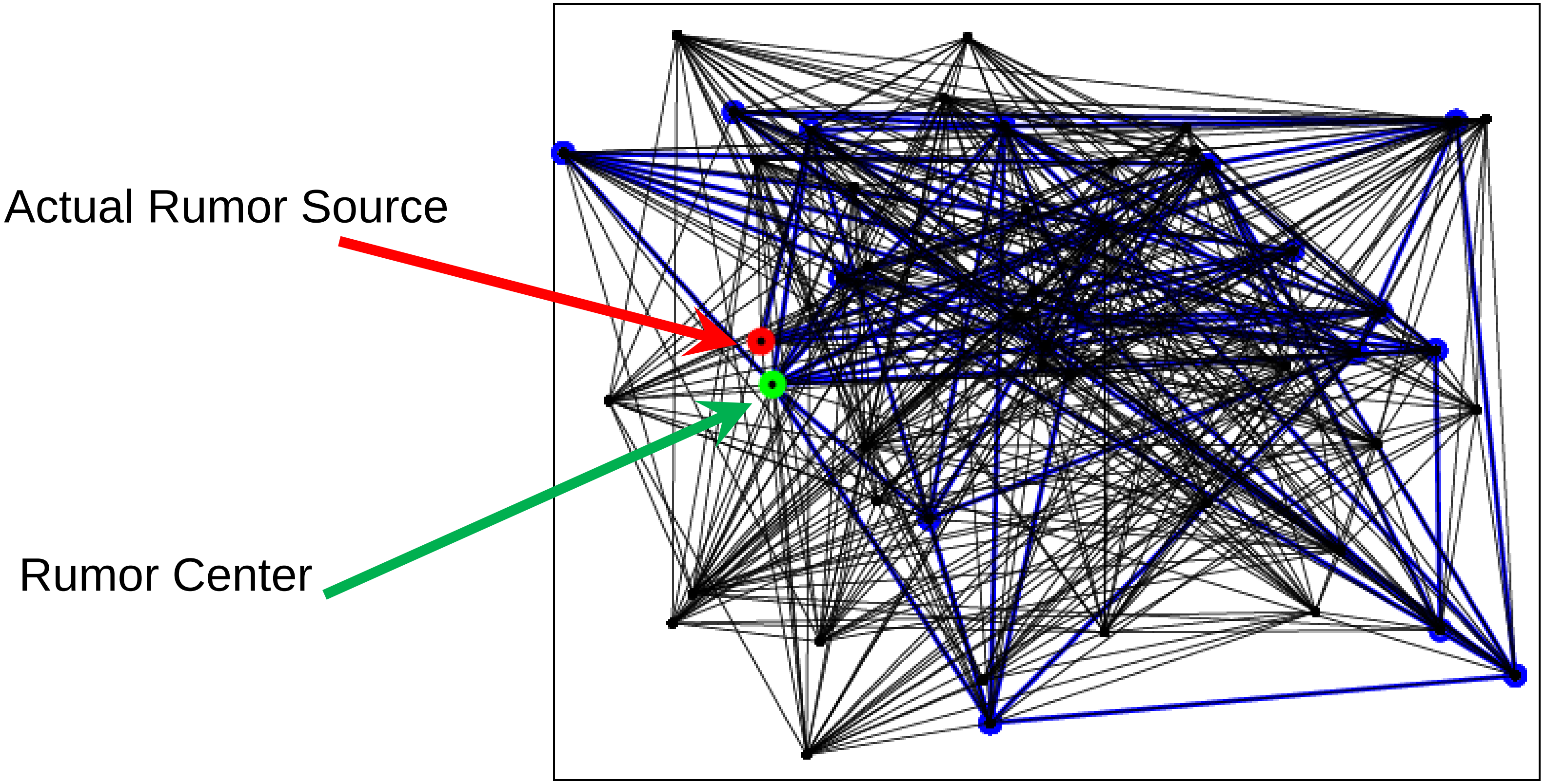
- Special case (Fully connected graph)

the degree of every nodes in the underlying graph G_N and the rumor graph G_N^K are respectively $N-1$ and $K-1$.

$$P(\sigma \mid s) = \prod_{k=2}^{K} \frac{k-1}{(K-1)(k-1) - 2C_{k-1}^2} = \frac{1}{(K-1)!}. \qquad P(G_N^K \mid s) = (K-1)!P(\sigma \mid s) = 1.$$

Detection for General Graph

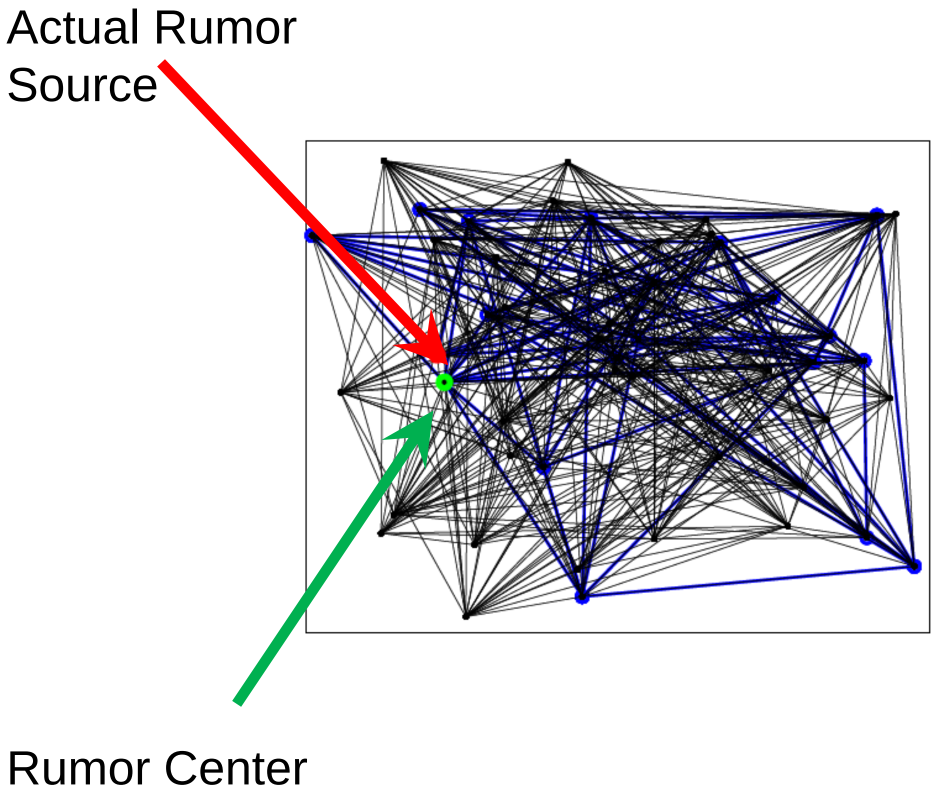
- Erdos-Renyi random graph (*N=50, K=20, p=0.4*)

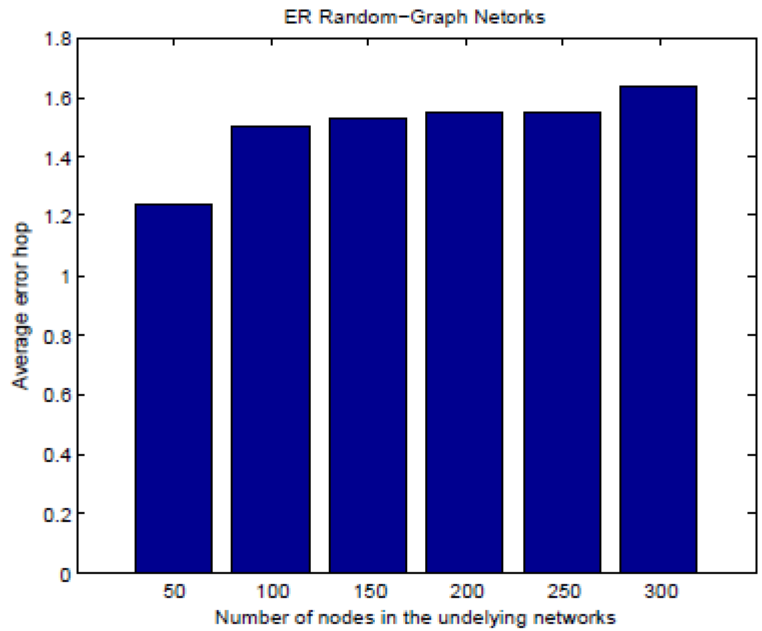


1) Start with N isolated nodes;
2) Add an edge between two nodes with probability p.

Detection for General Graph

- Erdos-Renyi random graph



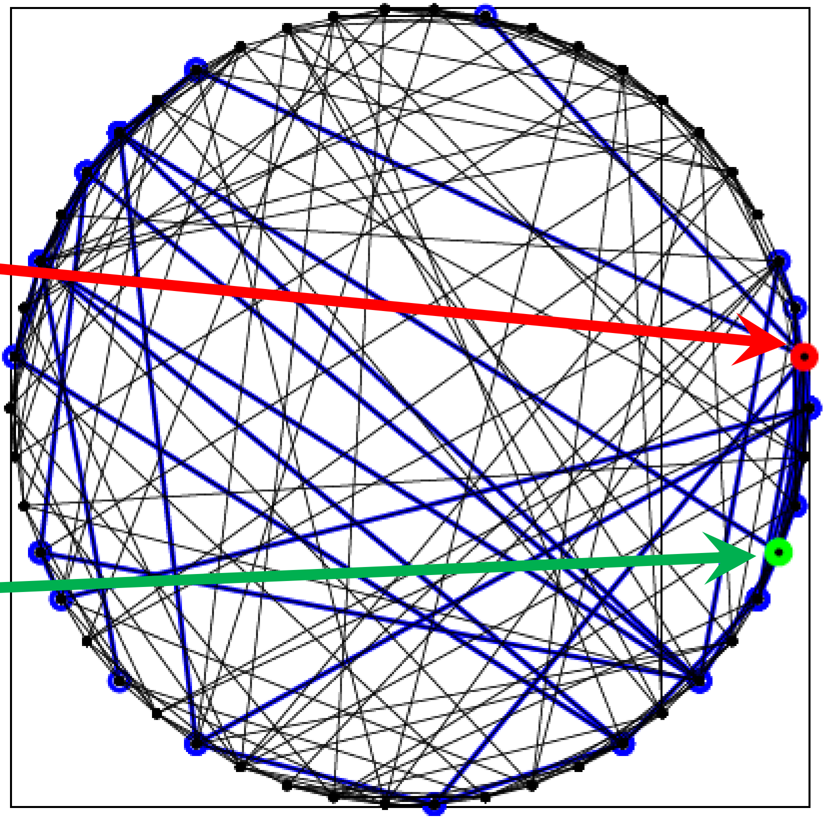


Detection for General Graph

- Newman-Watts small world (*N=50, K=20, m0=4, p=0.4*)

Actual Rumor Source

Rumor Center

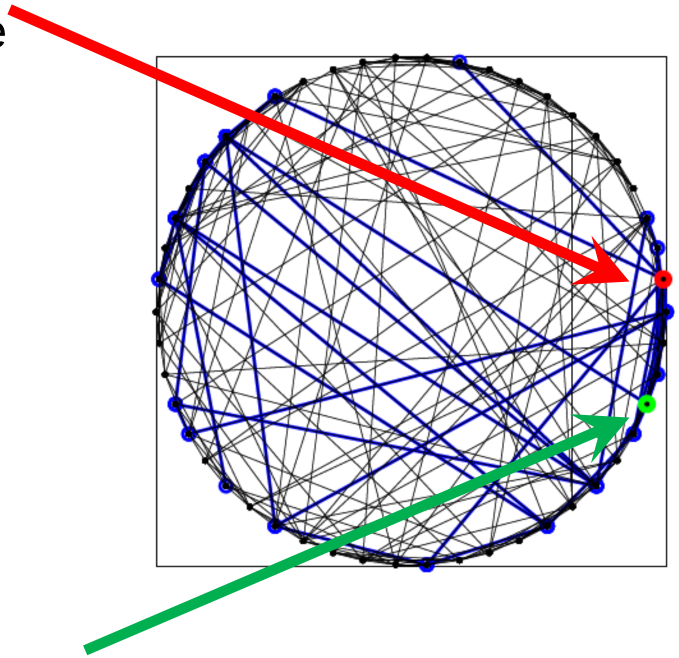


1) Start with a ring-shaped network with N nodes, in which each node is connected to its $2m_0$ neighbors, where $m_0 > 0$ is a (small) positive integer.
2) Add an edge between two nodes with probability p.

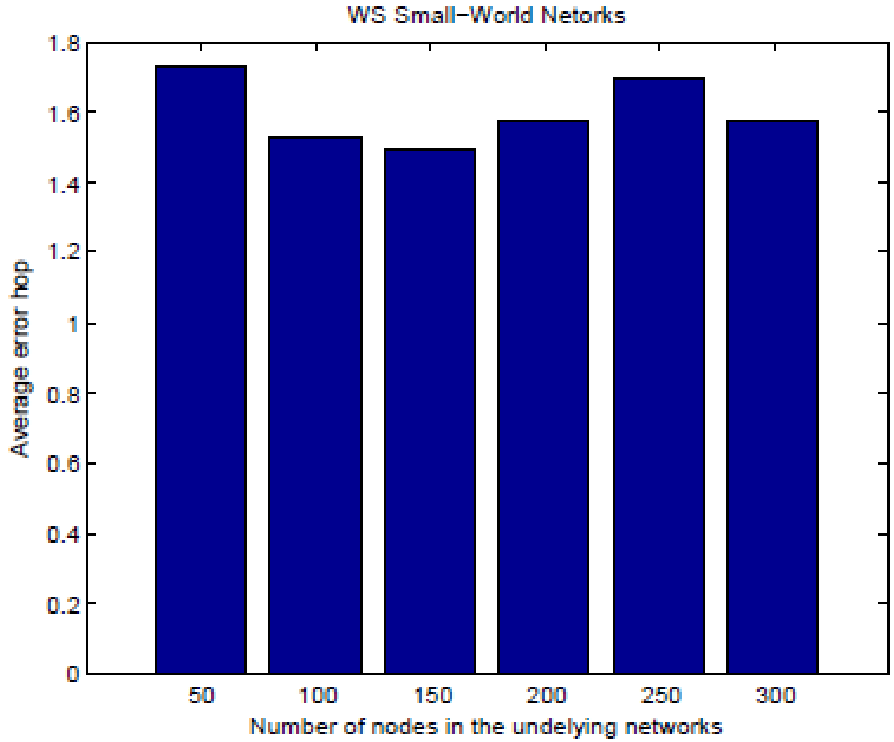
Detection for General Graph

- Newman-Watts small world graph

Actual Rumor Source

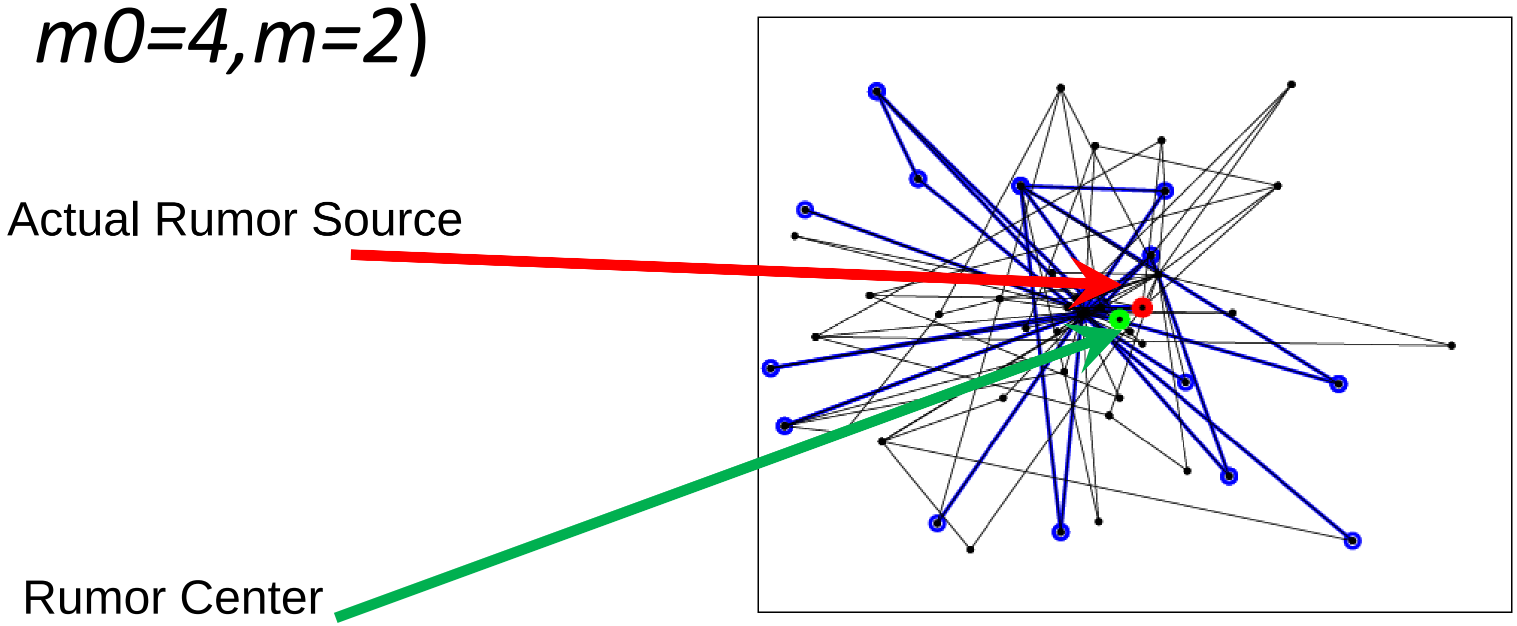


Rumor Center



Detection for General Graph

- Barabasi-Albert scale-free graph (*N=50, K=20, m0=4,m=2*)



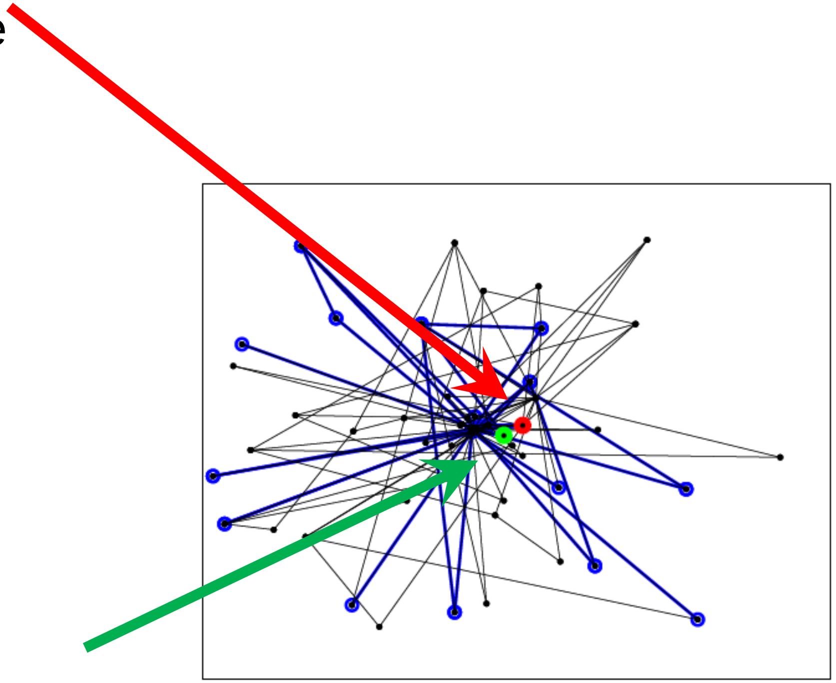
1) Growth: start with a small fully-connected network having $m_0 \geq 1$ nodes, and add one new node to the network each time by connecting to m existing nodes, where ($m \leq m_0$).
2) Preferential attachment: The new node is connected to an existing node i of degree d_i according to the following probability:

$$\Pi_i = \frac{d_i}{\sum_{j=1}^{N} d_j}.$$

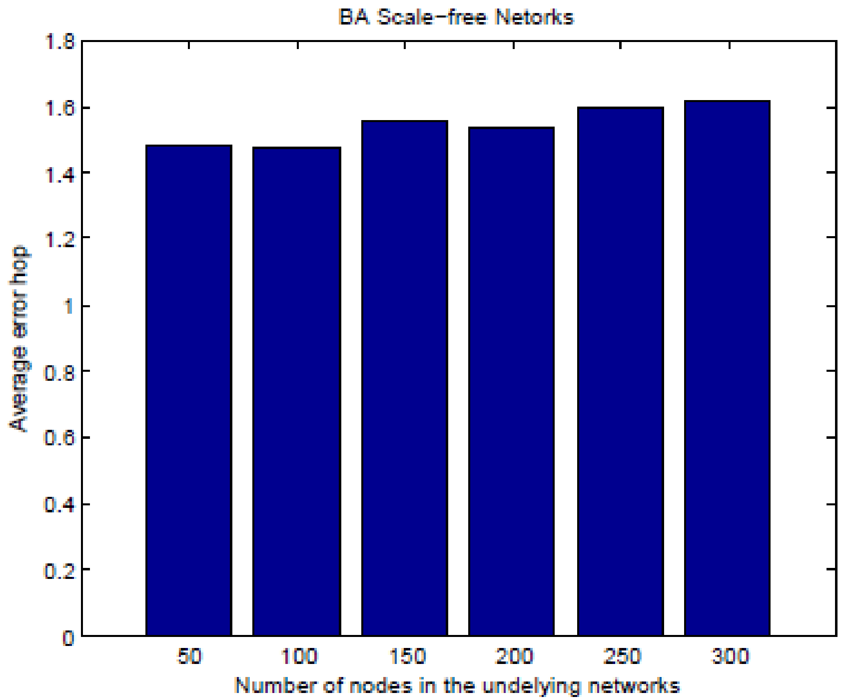
Detection for General Graph

- Barabasi-Albert scale free graph

Actual Rumor Source



Rumor Center



Extensions

- **Detection with Multiple Observations**
  - **How many observations to take?**
  - **2 Observations more than double the detectability performance of a single observation**

Zhao, Dong, Zhang and Tan
ACM SIGMETRICS 2014

- **Open Questions**
  - **General graph detection**
  - **How good or how bad is the Breadth-first Search Heuristic?**

Extensions

- Centrality in Social Network Analysis
  - Degree
  - Distance
  - Betweenness
  - Eigenvector
  - …etc
- Center in Graph Theory
  - Mass, Jordan
- General unifying link still missing
  - Jordan center not equal to mass center in general

Data Mining and Network Forensics

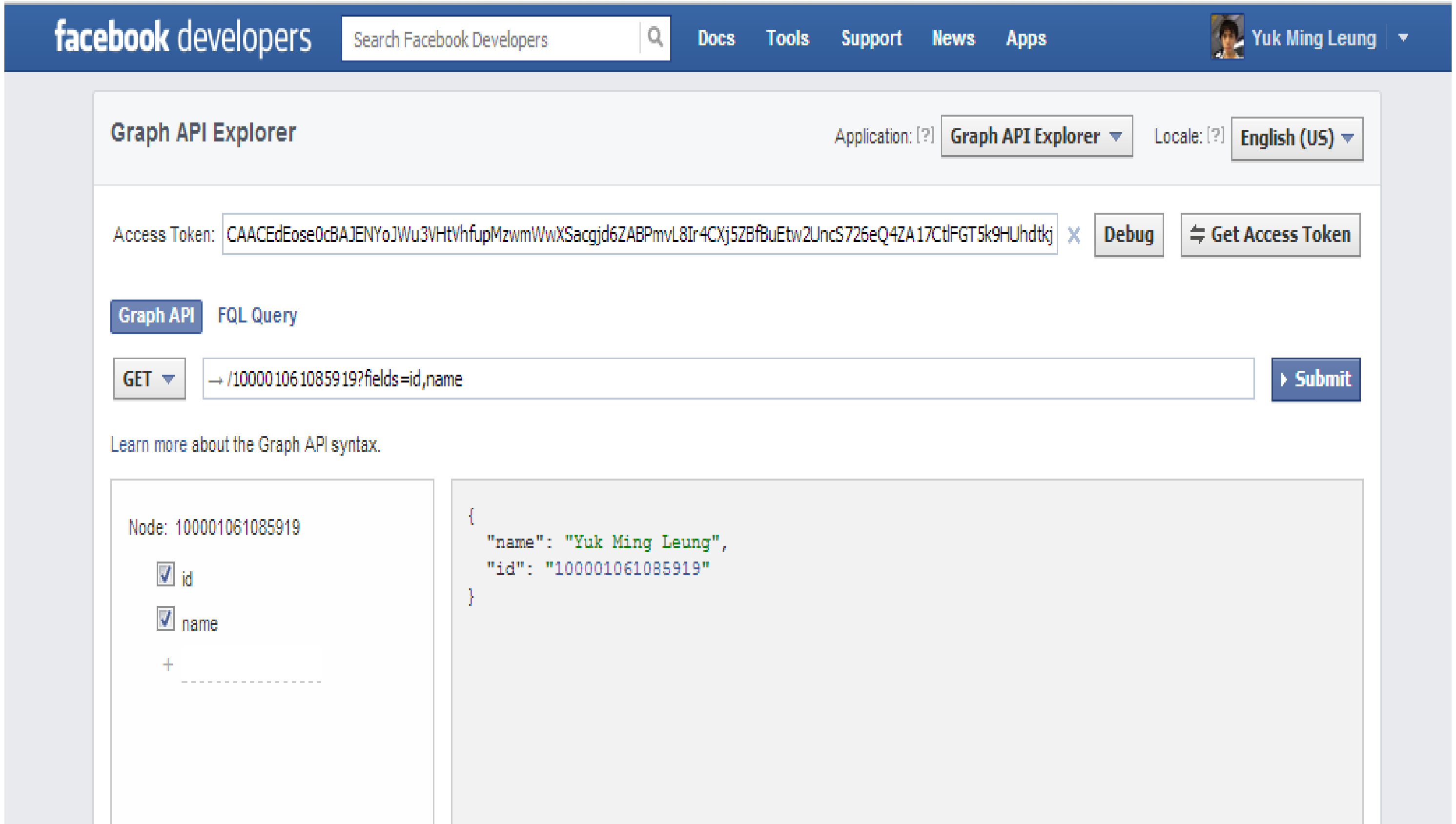
- Forensics in Online social networks

Data Mining and Network Forensics

- Many interactions over online social networks that are recorded and available from API service to glimpse social relationship of users
- Provide clues for rumor source detection and other cyber-security forensics algorithms
- How to even obtain a single snapshot observation of the graph?
- Bridge deep gulf between theory and practice
  - There is nothing more practical than a good theory!

Facebook Graph

- Enables application to read/write on Facebook social graph



Facebook Graph

- Example API call (in Javascript)

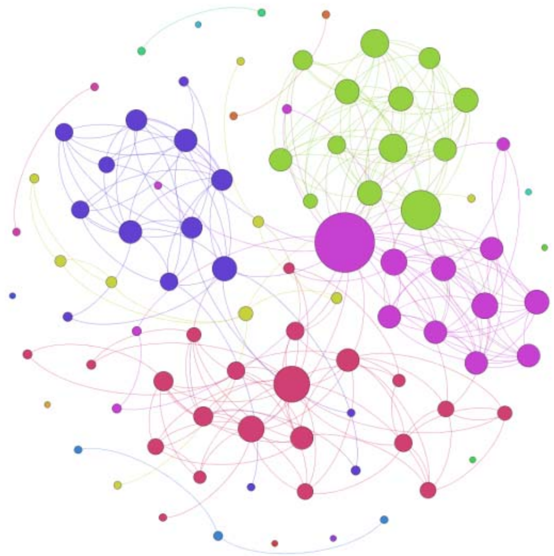
```
FB.api(
    "/me/friends?fields=id",
    function (response) {
      if (response &&
!response.error) {
        /* handle the result */
      }
    }
);
```

Facebook Graph

- API calls require access token
  - For identification of users, apps etc,
  - For granting permission, web login etc
- Facebook query language
  - Query data from the Graph API (SQL-style interface)

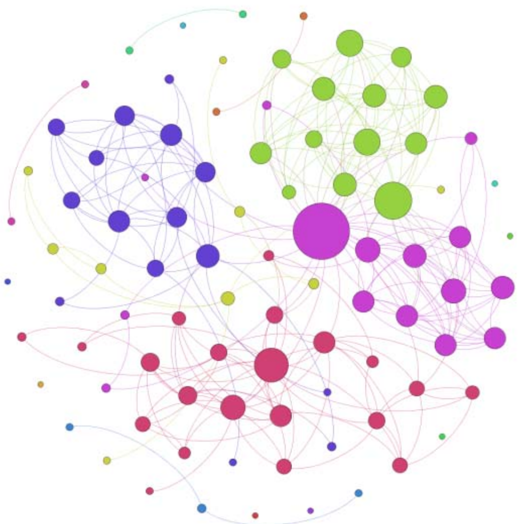
e.g.

```
    SELECT uid1, uid2
FROM friend   where
    uid1 in ' + user1 + '
AND uid2 in ' +    user2
+ ' AND uid1 < uid2';
```



Data Mining

- Rate-constrained data scraping
- Access control for privacy and security
- How to use semantics to infer possessing a rumor?
- How to link social graph with technological graph?



Conclusion

- **Rumor Centrality**
  - **Center of a Network**
- **Network features: Suspects, Connectivity, Observations**
- **Detectability and Detection**
  - **Statistical inference, probability theory, graph theory, Information theory**
  - **Scalable algorithms**
- **Numerous Open Issues:**

– Heterogeneous connectivity and spreading models

– Real-world data traces

– Practical network forensics protocol in online social networks

Thank You

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SI Spreading Model