Wireless Network Economics and Games

Jianwei Huang

Network Communications & Economics Lab
The Chinese University of Hong Kong
ncel.ie.cuhk.edu.hk

References

- J. Huang and L. Gao, "Wireless Network Pricing," Synthesis Lectures on Communication Networks (Series Editor J. Walrand), Morgan & Claypool, July 2013, http://jianwei.ie.cuhk.edu.hk/publication/Book/WirelessNetworkPricing.pdf

- J. Huang, "How Do We Play Games?" an online short course, https://itunes.apple.com/hk/course/how-do-we-play-games/id642100914

Why Wireless Economics and Games?

Wireless Utopia

- Wireless spectrum is unlimited
- Wireless communication is fast and reliable
- Heterogeneous wireless technologies co-exist in harmony
- Wireless users have reasonable data needs
- Wireless providers maximize social welfare

Wireless Reality

- Wireless spectrum is ~~unlimited~~ very limited
- Wireless communication is ~~fast and reliable~~ is slow and unreliable
- Heterogeneous wireless technologies ~~co-exist in harmony~~ compete and interfere with each other
- Wireless users have ~~reasonable~~ exploding data needs
- Wireless providers maximize ~~social welfare~~ profits

How Economics can Help?

- Match wireless supply and demand
- Limited spectrum vs. new wireless services
  - Spectrum allocation and auction
  - Secondary spectrum markets
- Limited cellular capacity vs. growing data demands
  - Smart data pricing
  - Wi-Fi data offloading

Tech-Econ Coupling

- Different technology characteristics
  - Cellular vs. Wi-Fi: coverage, data rate, and cost
- Distributed and heterogeneous networks
  - Different operators have different interests
  - Sophisticated devices capable of adaptation and optimization
- New technology adoption and evolution
  - Cellular technology upgrade (3G -> 4G)
  - Skype Wi-Fi adoption

Tutorial Outline

- Theory
  - Game theory
  - Economics
- Applications
  - Technology background and problem formulation
  - Key economics and game methodologies

Theory

Theory Outline

- Game theory:
    - Static games
    - Dynamic games
- Economics:
    - Price discrimination
    - Network Externality

Game Theory: Static Games

Prisoner's dilemma

- Two suspects are arrested.

Prisoner's dilemma

- Two suspects are arrested.
- The police lack sufficient evidence to convict the suspects, unless at least one confesses.

Prisoner's dilemma

- Two suspects are arrested.
- The police lack sufficient evidence to convict the suspects, unless at least one confesses.
- The police hold the suspects in separate rooms, and tell each of them three possible consequences.

Prisoner's dilemma

- If both deny: 1 month in jail each.

Prisoner's dilemma

- If both deny: 1 month in jail each.
- If both confess: 6 months in jail each.

Prisoner's dilemma

- If both deny: 1 month in jail each.
- If both confess: 6 months in jail each.
- If one confesses and one denies
  - The one confesses: walk away free of charge.
  - The one denies: serve 12 months in jail.

Prisoner's dilemma

		Player 2	
		Deny	Confess
Player 1	Deny	-1, -1	-12, 0
	Confess	0, -12	-6, -6

Prisoner's dilemma

		Player 2
		Deny
Player 1	Deny	-1, -1
	Confess	0, -12

Prisoner's dilemma

Player 2

Player 1		Confess
	Deny	-12, 0
	Confess	<u>-6</u>, -6

Strictly Dominant

- Confess is a **strictly dominant strategy** for player 1,
- It always leads to the best payoff, independent of player 2's strategy.

Prisoner's dilemma

		Player 2: Deny	Player 2: Confess
Player 1	Deny	-1, -1	-12, 0
	Confess	0, -12	-6, -6

PRISONER'S DILEMMA

Player 2

Player 1

	Deny	Confess
Deny	-1, -1	-12, 0

Prisoner's dilemma

Player 2

Player 1		Deny	Confess
	Confess	0, -12	-6, <u>-6</u>

Strictly Dominant

- Confess is also a **strictly dominant strategy** for player 2.

Prisoner's dilemma

		Player 2	
		Deny	Confess (dominant)
Player 1	Deny	-1, -1	-12, 0
	Confess (dominant)	0, -12	-6, -6

Prisoner's dilemma

		Player 2	
		Deny	Confess (dominant)
Player 1	Confess (dominant)	0, -12	-6, -6

Prisoner's dilemma

Player 2

Confess
(dominant)

Player 1

Confess
(dominant)

-6, -6

Prisoner's dilemma

		Player 2	
		Deny	Confess (dominant)
Player 1	Deny	-1, -1	-12, 0
	Confess (dominant)	0, -12	-6, -6

Prisoner's dilemma

- Prediction of the game: (confess, confess)
- Dilemma:
  - (confess, confess) leads to a payoff of (-6, -6)
  - (deny, deny) leads to a payoff of (-1, -1)
- Key reason: selfish optimization.

Finding Equilibrium

- When there are no strictly dominant strategies, we can not easily "reduce" the game.
- Similar analysis: derive the best responses.
- A stable outcome (equilibrium) will be mutual best responses.

Stag Hunt

- Two hunters decide what to hunt without communications.
- Each one can hunt a stag (deer) or a hare.
- Successful hunt of stag requires cooperation.
- Successful hunt of hare can be done individually.
- Simultaneous decisions without prior communications.

Stag Hunt

Player 2

Player 1	Stag	Hare
Stag	5, 5	0, 2
Hare	2, 0	2, 2

Stag Hunt

- There is no strictly dominant or strictly dominated strategies.
- We will find out a player's **best response** given the other player's choice.

Stag Hunt

		Player 2	
		Stag	Hare
Player 1	Stag	5, 5	0, 2
	Hare	2, 0	2, 2

Stag Hunt

Player 2

Player 1		Stag
	Stag	5, 5
	Hare	2, 0

Stag Hunt

Player 2

		Stag
Player 1	Stag	5, 5
	Hare	2, 0

Stag Hunt

Player 2

Player 1

		Hare
Player 1	Stag	0, 2
	Hare	2, 2

Stag Hunt

Player 2

Player 1		Hare
	Stag	0, 2
	Hare	2, 2

Stag Hunt

		Player 2	
		Stag	Hare
Player 1	Stag	$\underline{5}$, 5	0, 2
	Hare	2, 0	$\underline{2}$, 2

Stag Hunt

		Player 2	
		Stag	Hare
Player 1	Stag	5, 5	0, 2
	Hare	2, 0	2, 2

Stag Hunt

		Player 2	
		Stag	Hare
Player 1	Stag	5, 5	0, 2
	Hare	2, 0	2, 2

Nash Equilibrium (NE)

- A pair of strategies = **Nash Equilibrium (NE)**
  - If each player is choosing the best response given the other player's strategy choice.
- At a Nash equilibrium, no player can perform a profitable deviation unilaterally.

Equilibrium Selection

- How to choose between two Nash equilibria?
  - (Stag, Stag) is **payoff dominant**: both players get the best payoff possible.
  - (Hare, Hare) is **risk dominant**: minimum risk if player is uncertain of each other's choice.
- Many theories, open problem.

Battle of Sexes

- A couple decide where to go during Friday night without communications.
- Husband prefers to go and watch football.
- Wife prefers to go and watch ballet.
- Both prefer to stay together during the night.

Battle of Sexes

Wife

Husband		Football	Ballet
	Football	4, 2	0, 0
	Ballet	0, 0	2, 4

Battle of Sexes

		Wife: Football
Husband	Football	4, 2
	Ballet	0, 0

Battle of Sexes

		Wife
		Football
Husband	Football	<u>4</u>, 2
	Ballet	0, 0

Battle of Sexes

		Wife
		Ballet
Husband	Football	0, 0
	Ballet	2, 4

Battle of Sexes

Wife

		Ballet
Husband	Football	0, 0
	Ballet	2, 4

Battle of Sexes

		Wife	
		Football	Ballet
Husband	Football	$\underline{4}$, 2	0, 0
	Ballet	0, 0	$\underline{2}$, 4

Battle of Sexes

Wife

		Football	Ballet
Husband	Football	4, 2	0, 0

Battle of Sexes

		Wife	
		Football	Ballet
Husband	Football	4, <u>2</u>	0, 0

Battle of Sexes

		Wife	
		Football	Ballet
Husband	Ballet	0, 0	2, 4

Battle of Sexes

		Wife: Football	Wife: Ballet
Husband	Ballet	0, 0	2, 4

Battle of Sexes

		Wife	
		Football	Ballet
Husband	Football	$\underline{4}$, $\underline{2}$	0, 0
	Ballet	0, 0	$\underline{2}$, $\underline{4}$

Continuous Games

- Next we show a continuous game
- A player has continuous (infinite) choices

Cournot Competition

- Two firms competing in the same market.
- Each firm i chooses its production level q_i.
  - The cost of producing one product is c.
- Total products in the market is $Q = q_1 + q_2$.
- The market clearing price is $P(Q)=\max(a-Q,0)$.

Cournot Competition

- Each firm i wants to choose q_i to maximize his profit

$$\pi_i\left(q_i, q_j\right) = q_i\left[P(q_i + q_j) - c\right] = q_i\left[a - (q_i + q_j) - c\right]$$

Nash Equilibrium

- Assume the Nash equilibrium is (q_1^*, q_2^*).

Best Response

- For firm *i*, its best response for a given q_j

$$\max_{0 \leq q_i < \infty} \pi_i(q_i, q_j^*) = \max_{0 \leq q_i < \infty} q_i \left[a - (q_i + q_j^*) - c\right]$$

- The solution

$$q_i = \frac{1}{2}\left(a - q_j^* - c\right)$$

Nash Equilibrium

- So we have

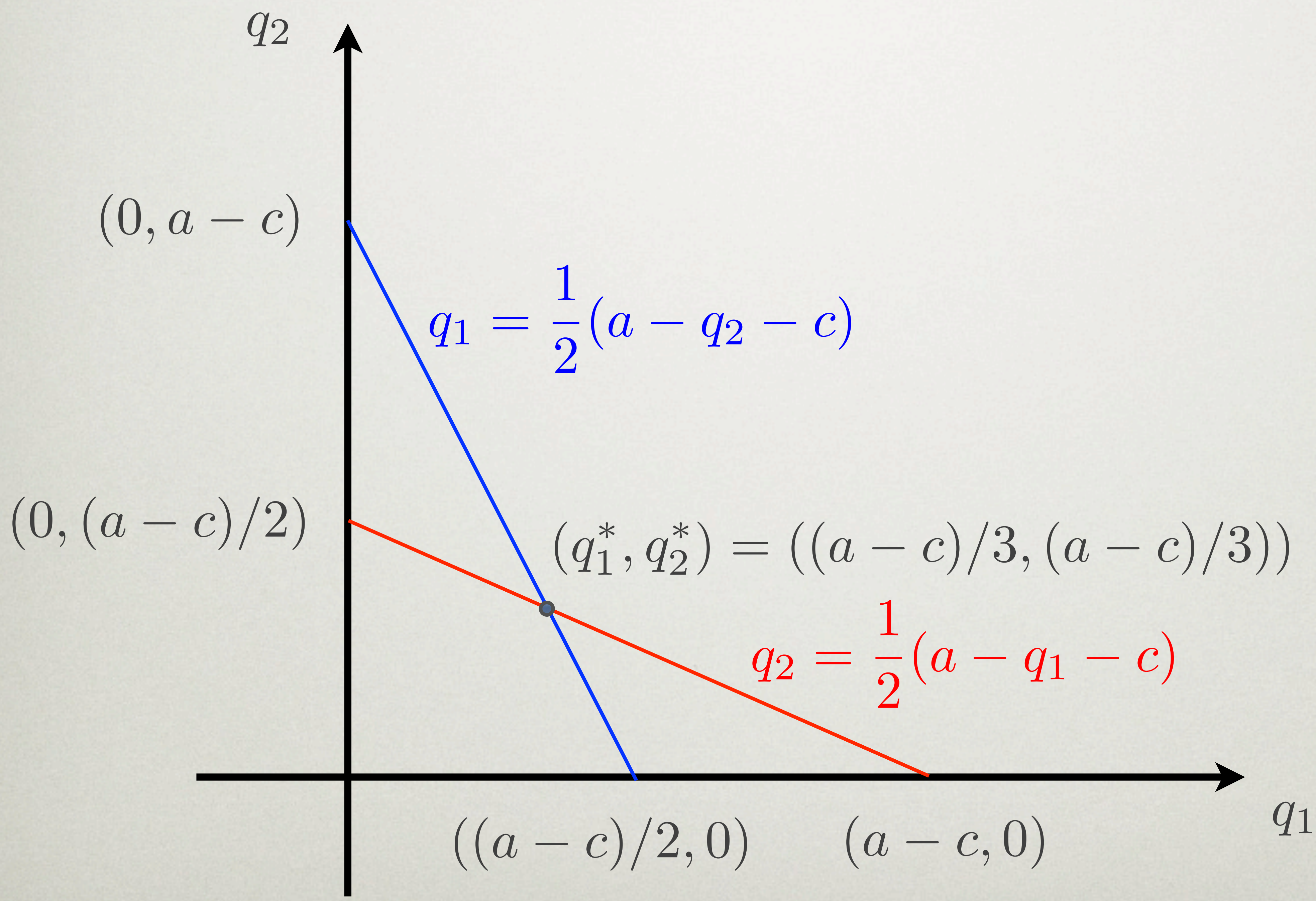
$$q_1^* = \frac{1}{2}(a - q_2^* - c)$$

$$q_2^* = \frac{1}{2}(a - q_1^* - c)$$

- This leads to the Nash equilibrium as

$$q_1^* = q_2^* = \frac{a-c}{3}.$$

Geometric Solution



Key Concepts Review

- Strictly dominate strategy
- Nash equilibrium
- Continuous games

Theory Outline

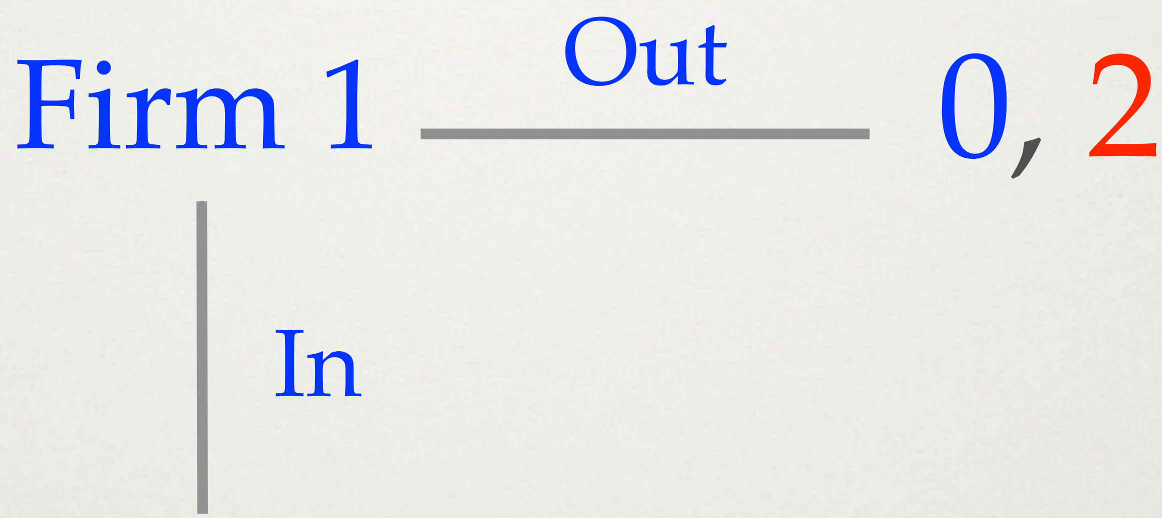
- Game theory:
  - Static games
  - Dynamic games
- Economics:
  - Price discrimination
  - Network Externality

Game Theory: Dynamic Games

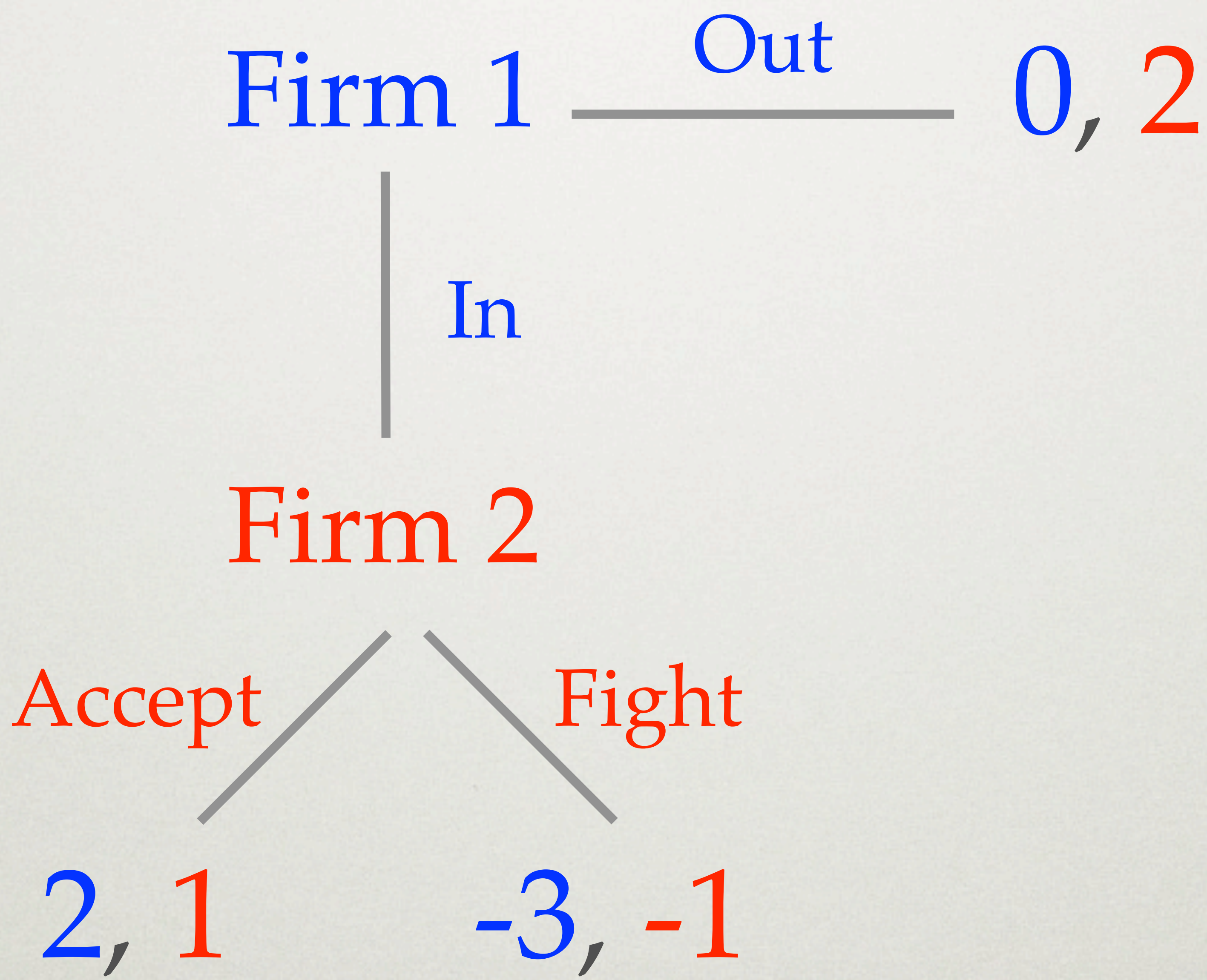
Market Entry

- Firm 1 is considering entering a market that currently has an incumbent (firm 2).
- Firm 1 can choose "In" or "Out".
  - If "Out", firm 1 gets nothing, and firm 2 enjoys monopoly.
- If "In", firm 2 can choose "Accept" or "Fight".
  - If firm 2 accepts, then firm 1 gets a larger market share due to a newer technology.
  - If firm 2 fights, then there is a price war and both firms get negative profits.

Market Entry



Market Entry



Market Entry

		Firm 2	
		Accept	Fight
Firm 1	Out	0, 2	0, 2
	In	2, 1	-3, -1

Market Entry

Firm 2

Firm 1		Accept
	Out	0, 2
	In	<u>2</u>, 1

Market Entry

Firm 2

Firm 1	Fight
Out	<u>0</u>, 2
In	-3, -1

Market Entry

Firm 1		Firm 2: Accept	Firm 2: Fight
	Out	0, 2	0, 2

Market Entry

Firm 2

Firm 1		Accept	Fight
	In	2, <u>1</u>	-3, -1

Market Entry

Firm 2

Firm 1	Accept	Fight
Out	0, <u>2</u>	<u>0</u>, <u>2</u>
In	<u>2</u>, <u>1</u>	-3, -1

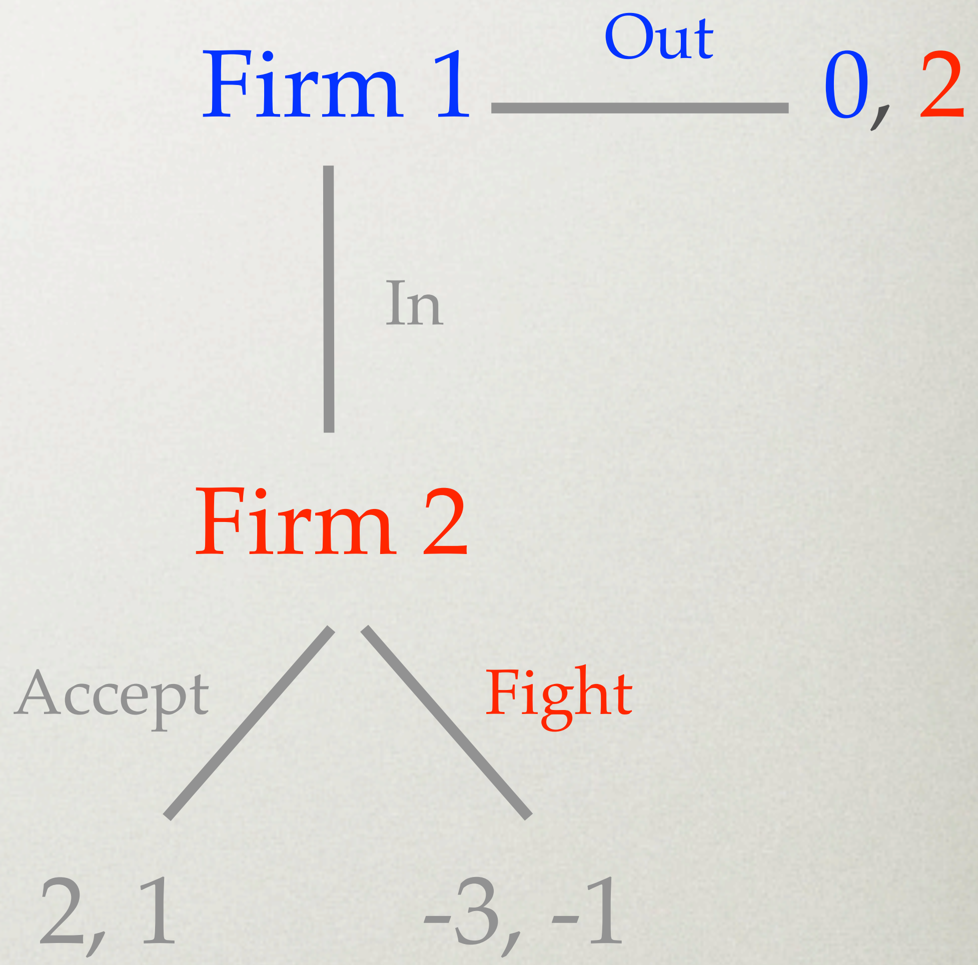
Market Entry

Firm 2

Firm 1		Accept	Fight
	Out	0, 2	0, 2
	In	2, 1	-3, -1

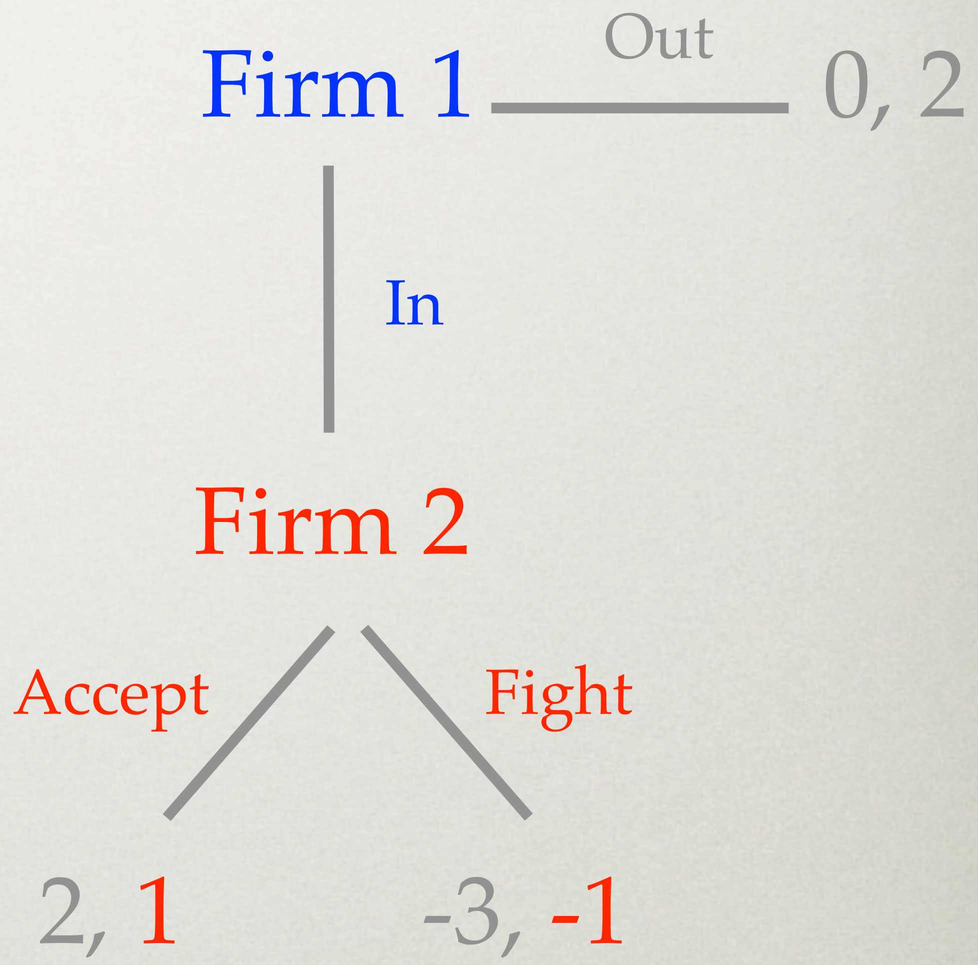
Market Entry

- Consider the Nash equilibrium (Out, Fight if entry occurs).
- Firm 1 chooses to stay Out because of firm 2's threat of Fight.



Non-credible Threat

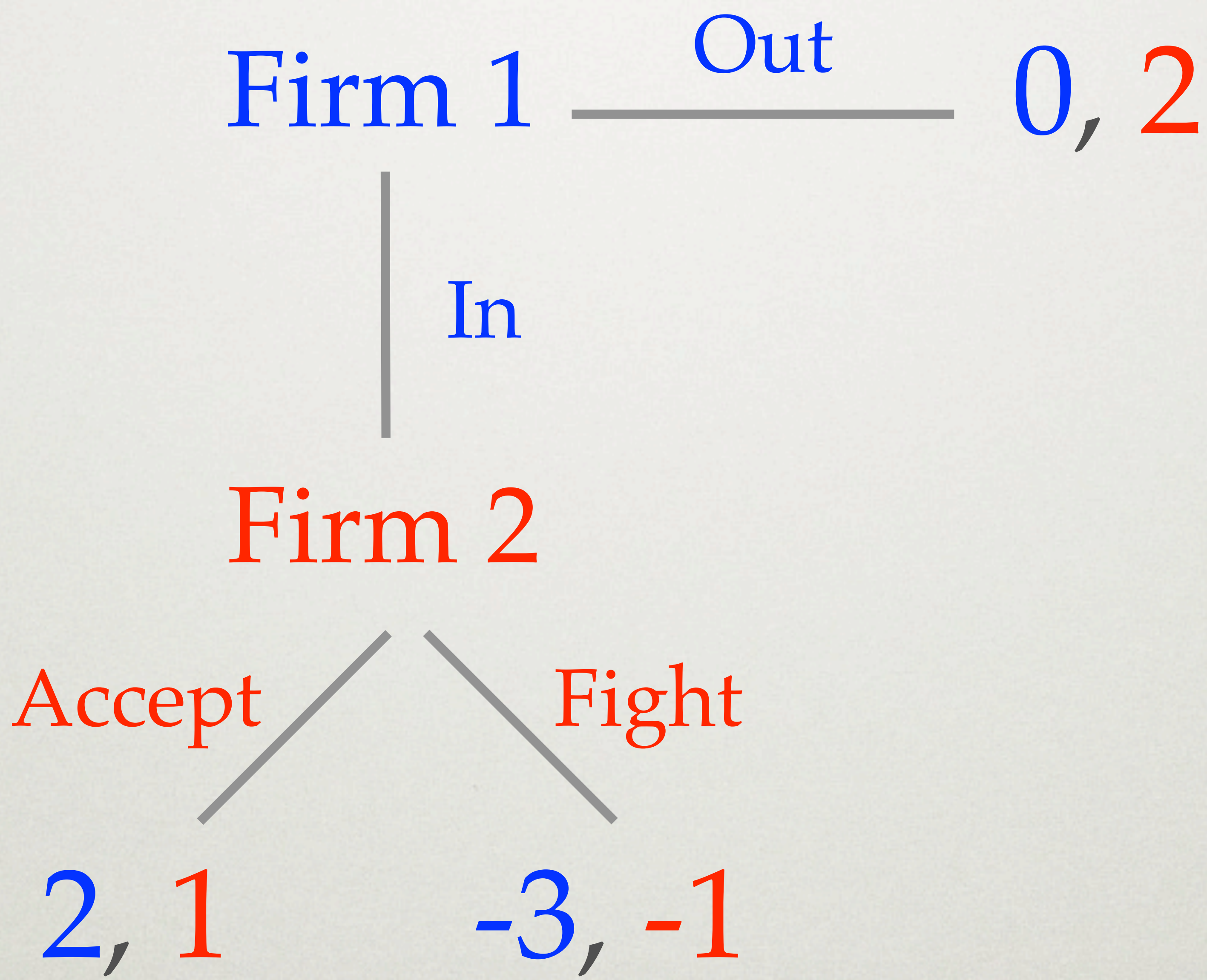
- However, if firm 1 chooses In, then firm 2 will actually choose to Accept instead.
- Hence Fight is a **non-credible threat**.



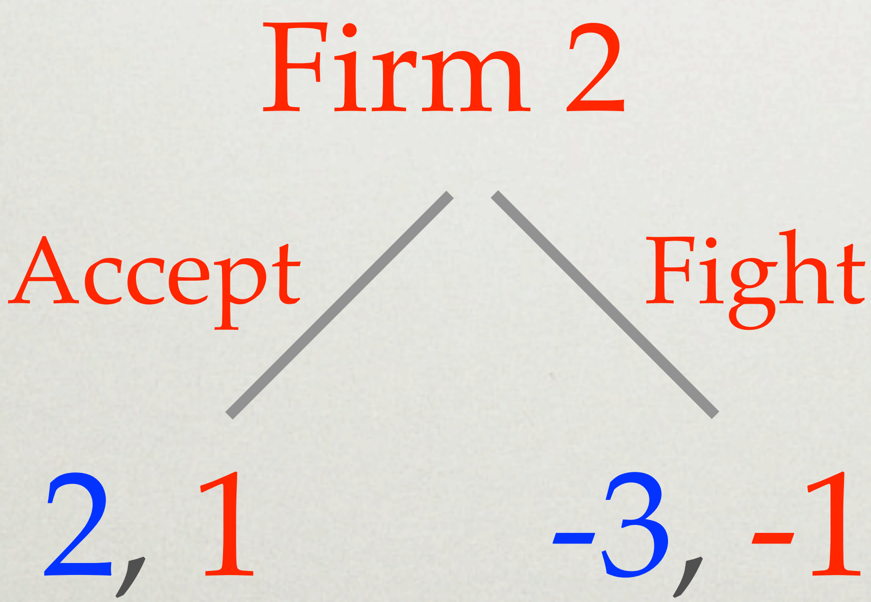
Equilibrium Refinement

- **Principle of sequential rationality**: an equilibrium strategy should be optimal at every point of the game tree.
- Examine each **subgame** through **backward induction**.

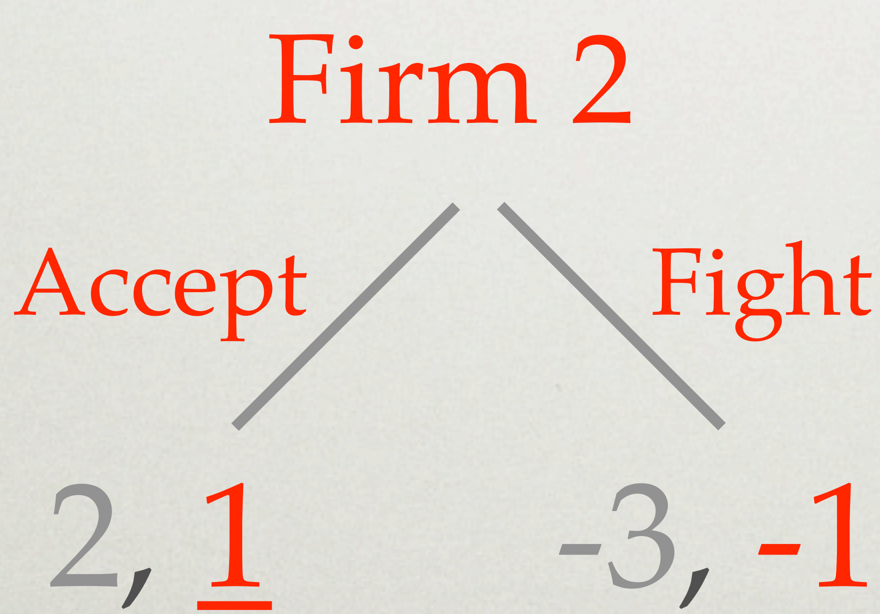
Subgame Analysis



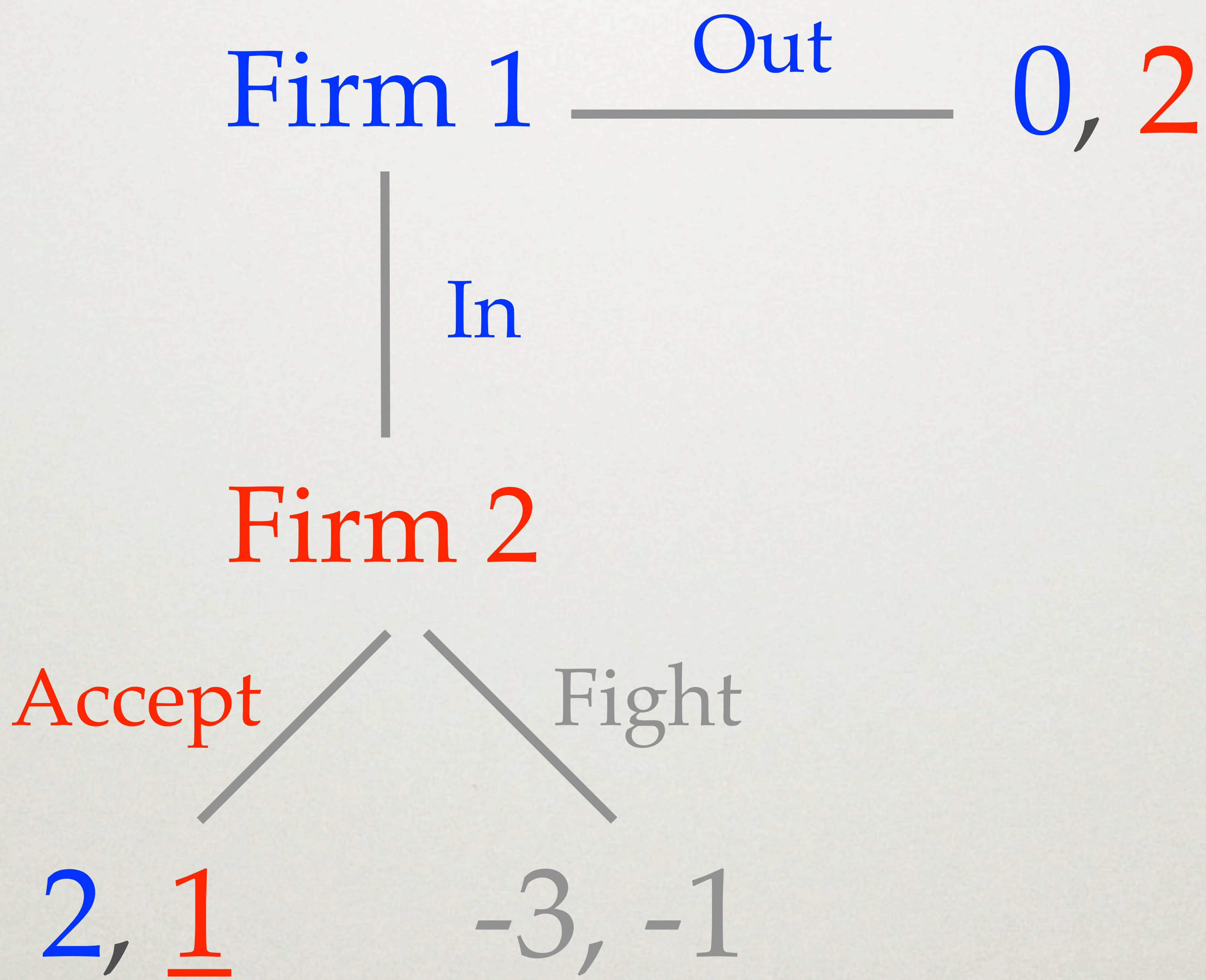
Subgame Analysis



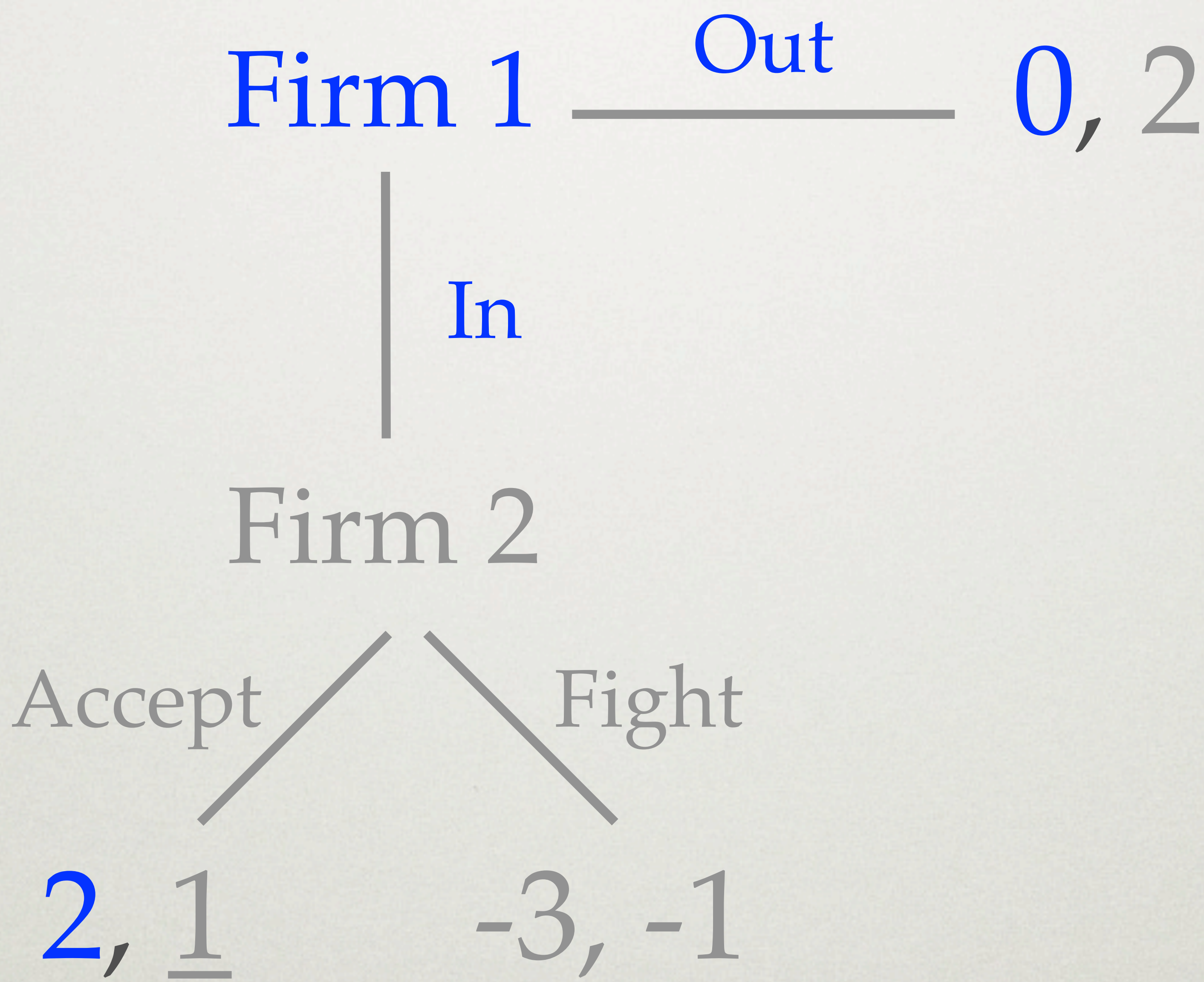
Subgame Analysis



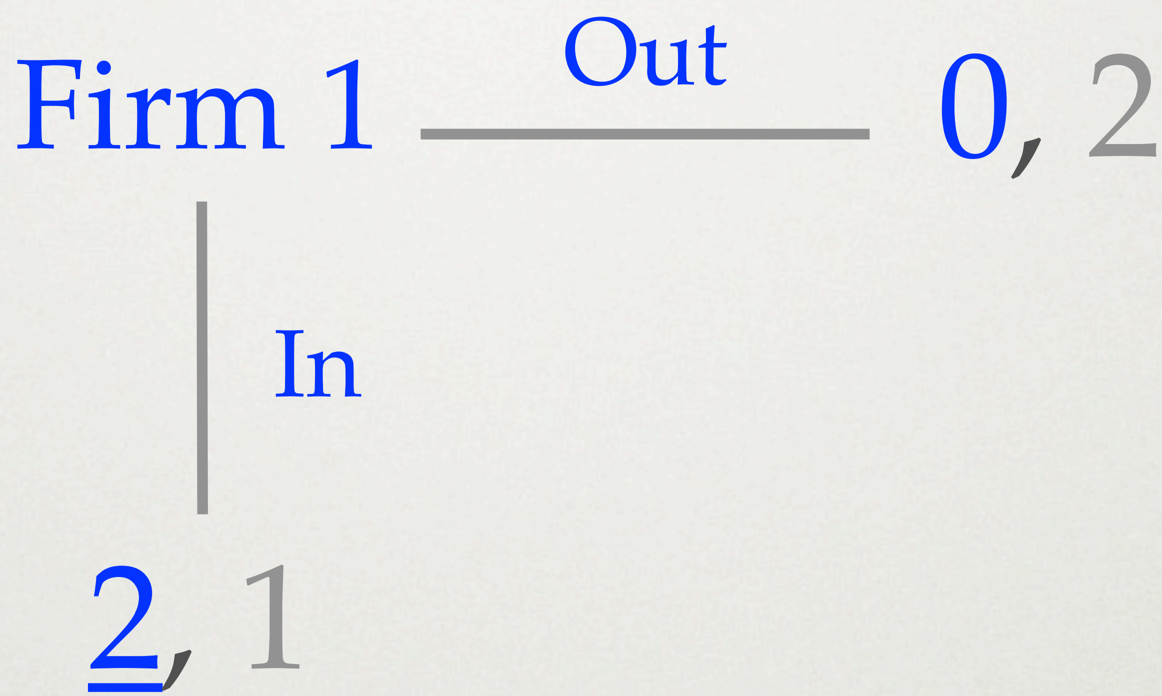
Subgame Analysis



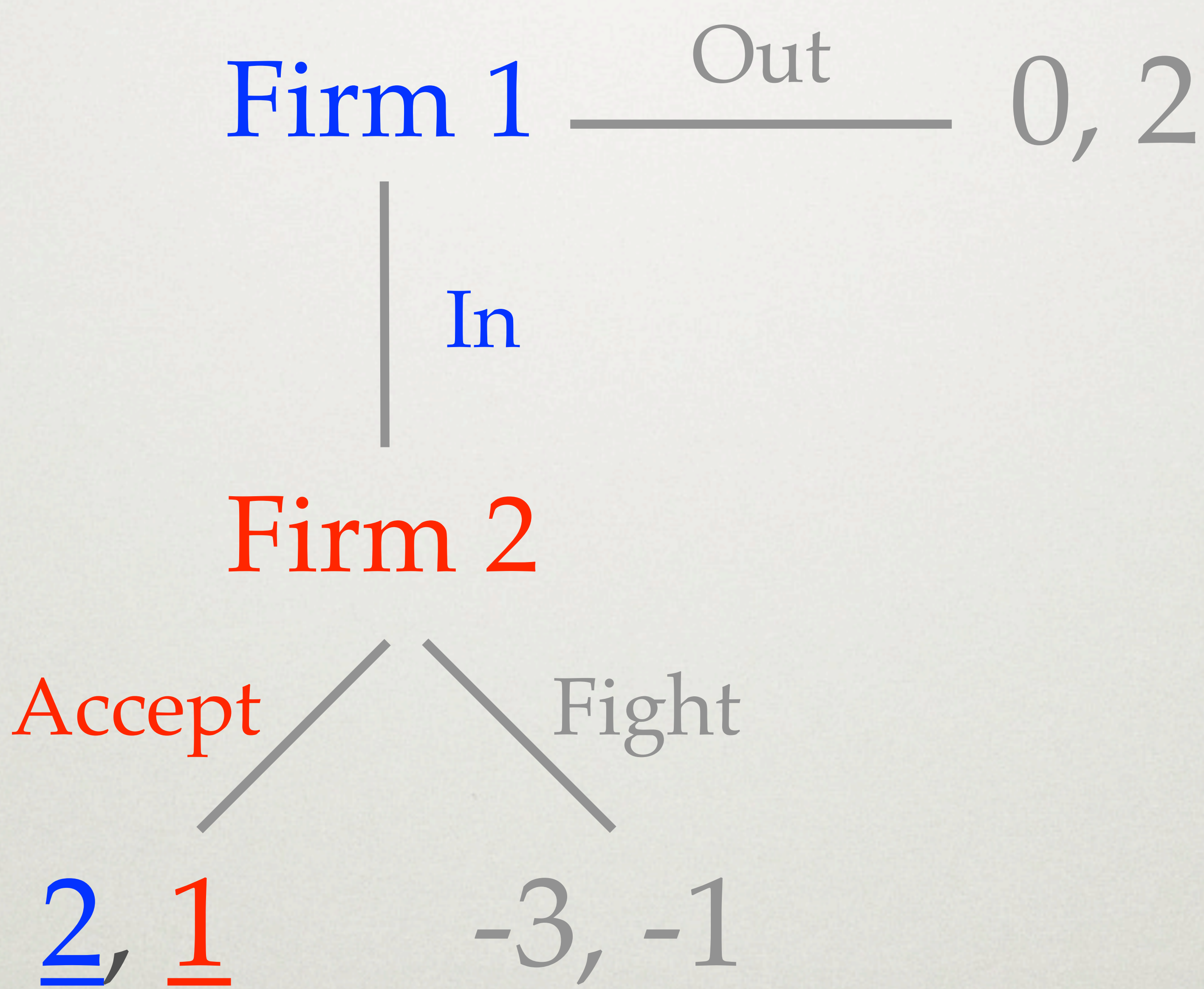
Subgame Analysis



Subgame Analysis



Equilibrium



Subgame Perfect Nash Equilibrium

- A strategy profile is a **subgame perfect Nash equilibrium (SPNE)** if it is a Nash equilibrium of every subgame of the original game.
- For market entry game, the unique SPNE is

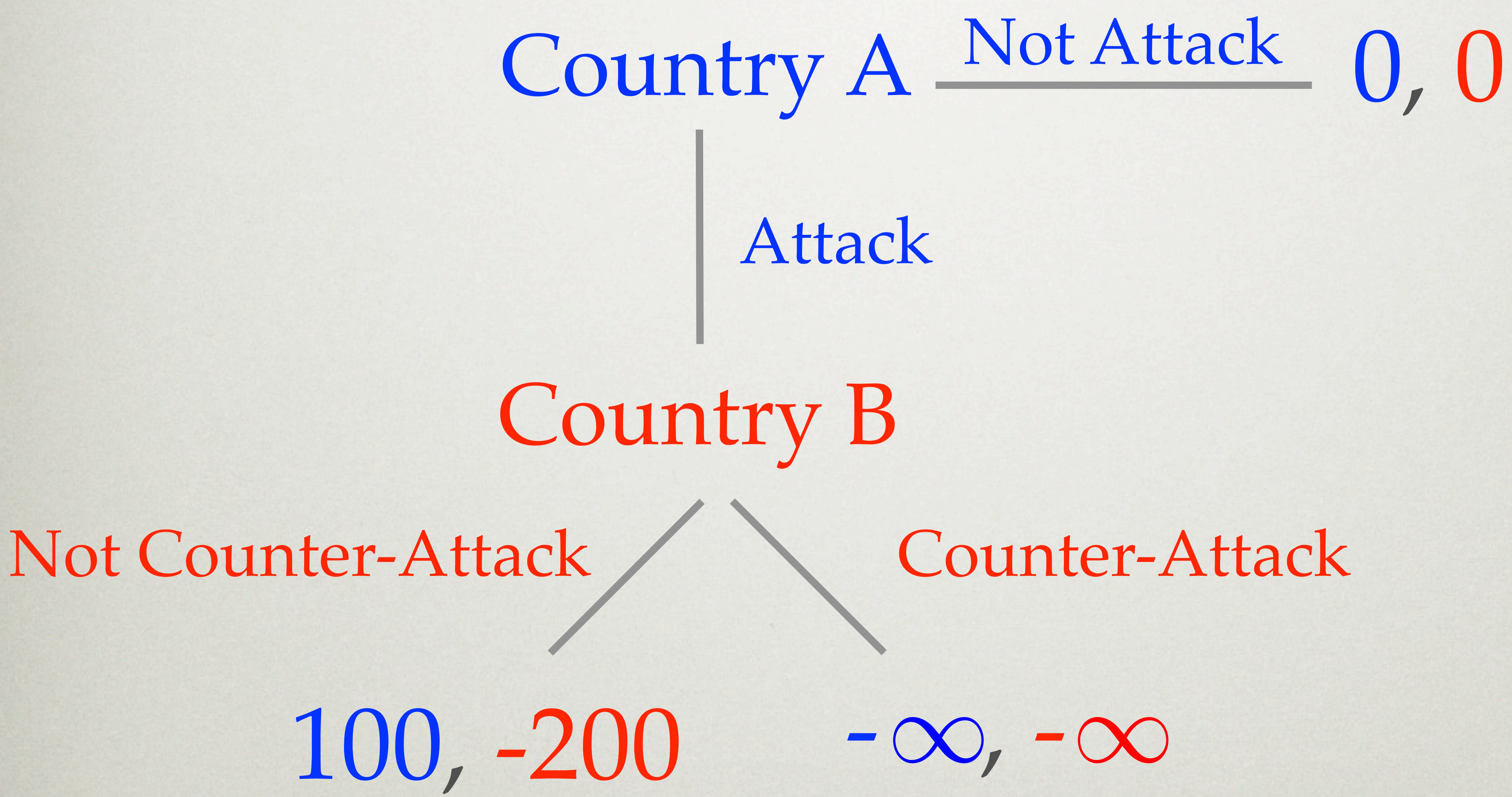
  (In, Accept if entry occurs).

Credible Threat

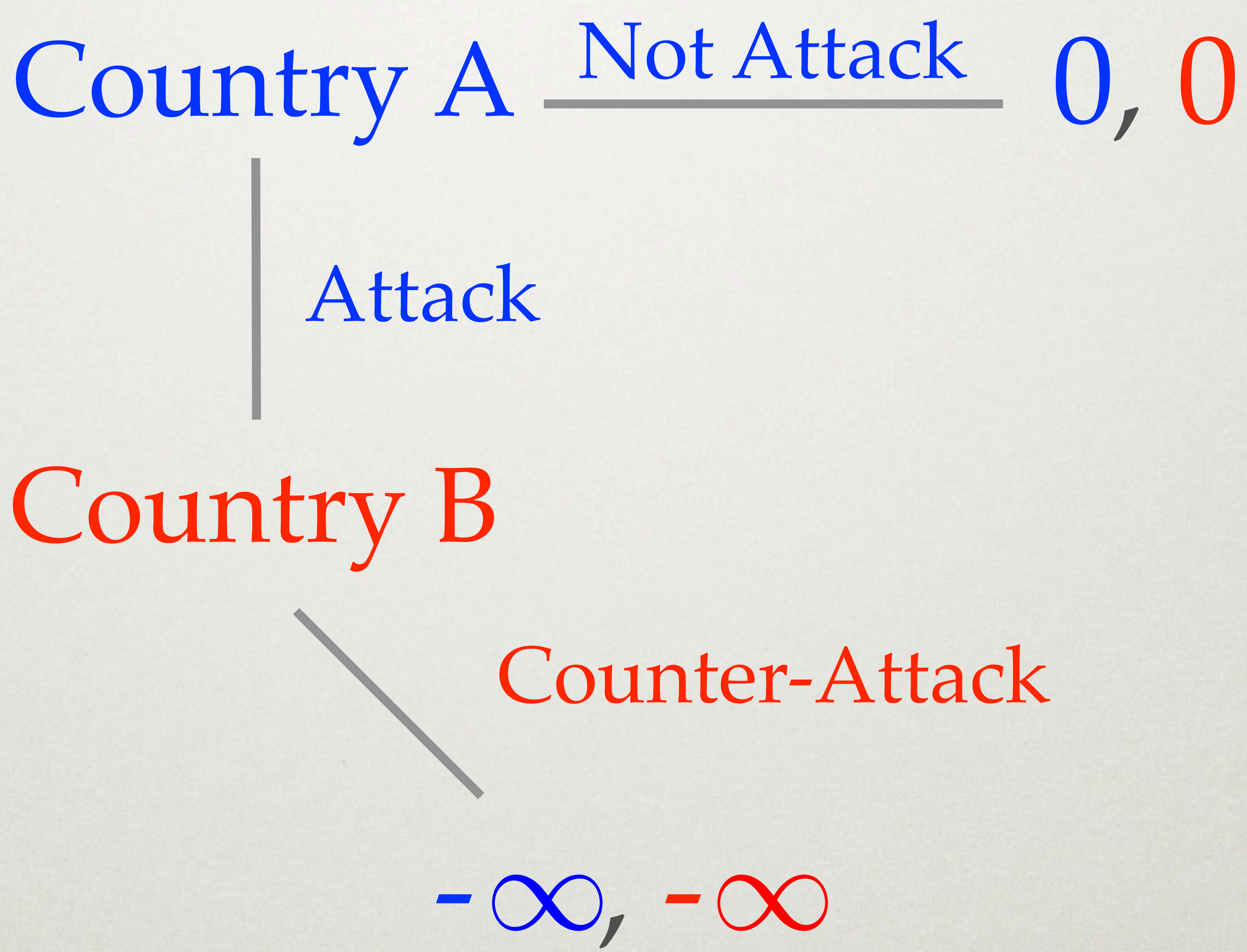
- How to make credible threat?
- Eliminate choices.


MOVIEWALLPAPERS.NET
Dr. Strangelove
Or:
How
I Learned
To
Stop
Worrying
And
Love
The
Bomb

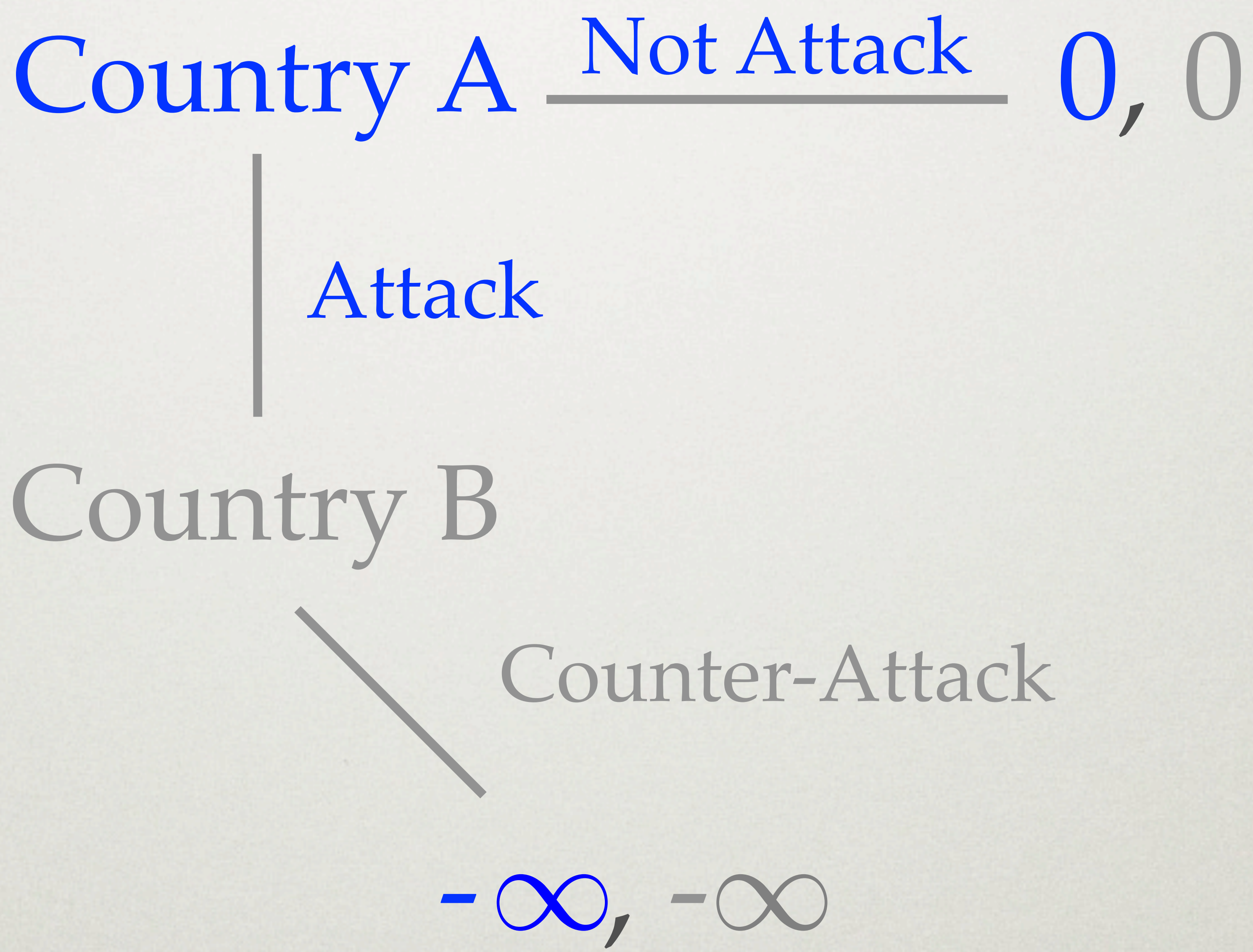
Dr. Strangelove



Dr. Strangelove

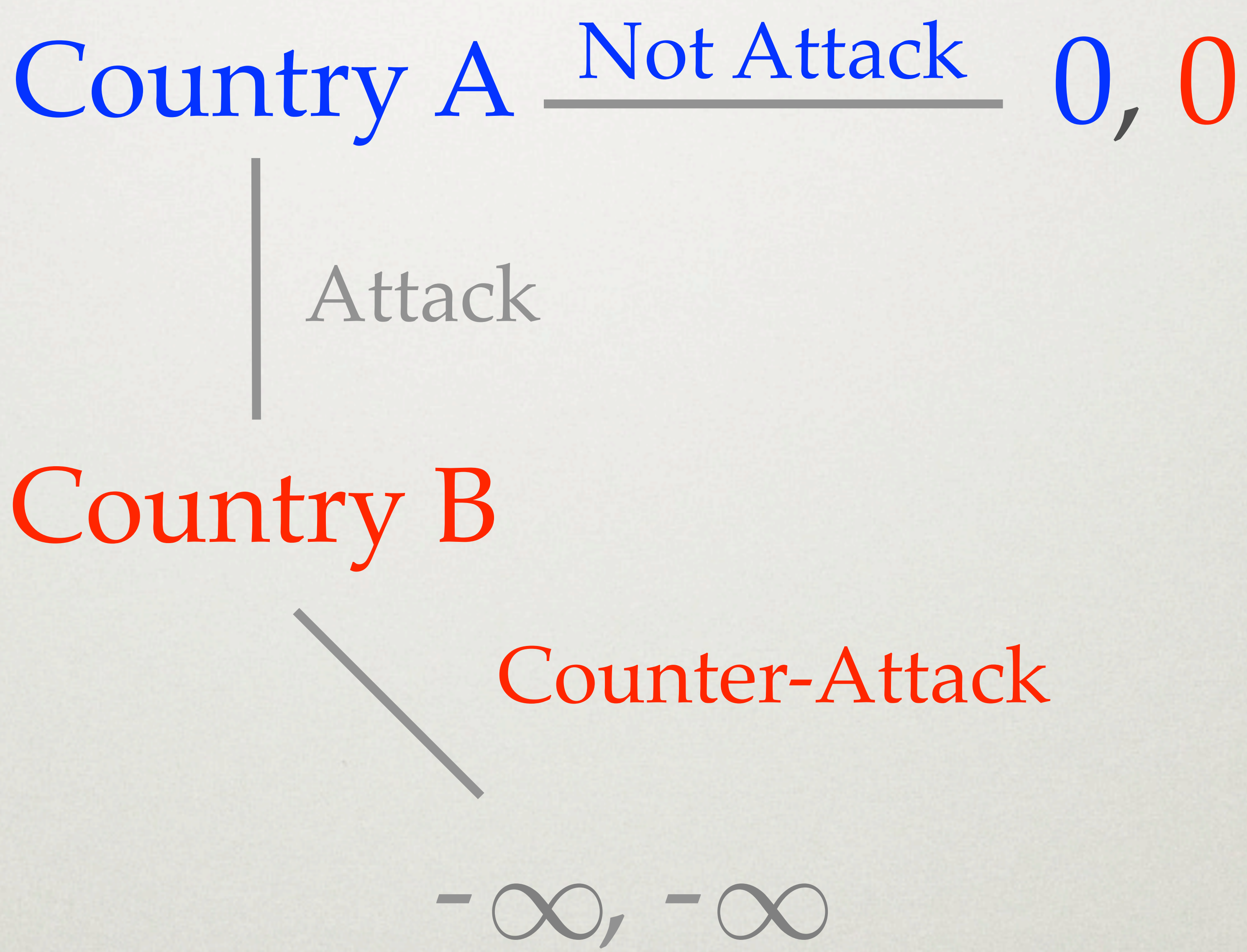


Dr. Strangelove



Dr. Strangelove

Dr. Strangelove



SPNE

- The unique SPNE of the Dr. Strangelove game is (Not Attack, Counter-Attack if Country A attacks).

First Mover Advantage

- Let us look at how the first mover can have an advantage.

Battle of Sexes

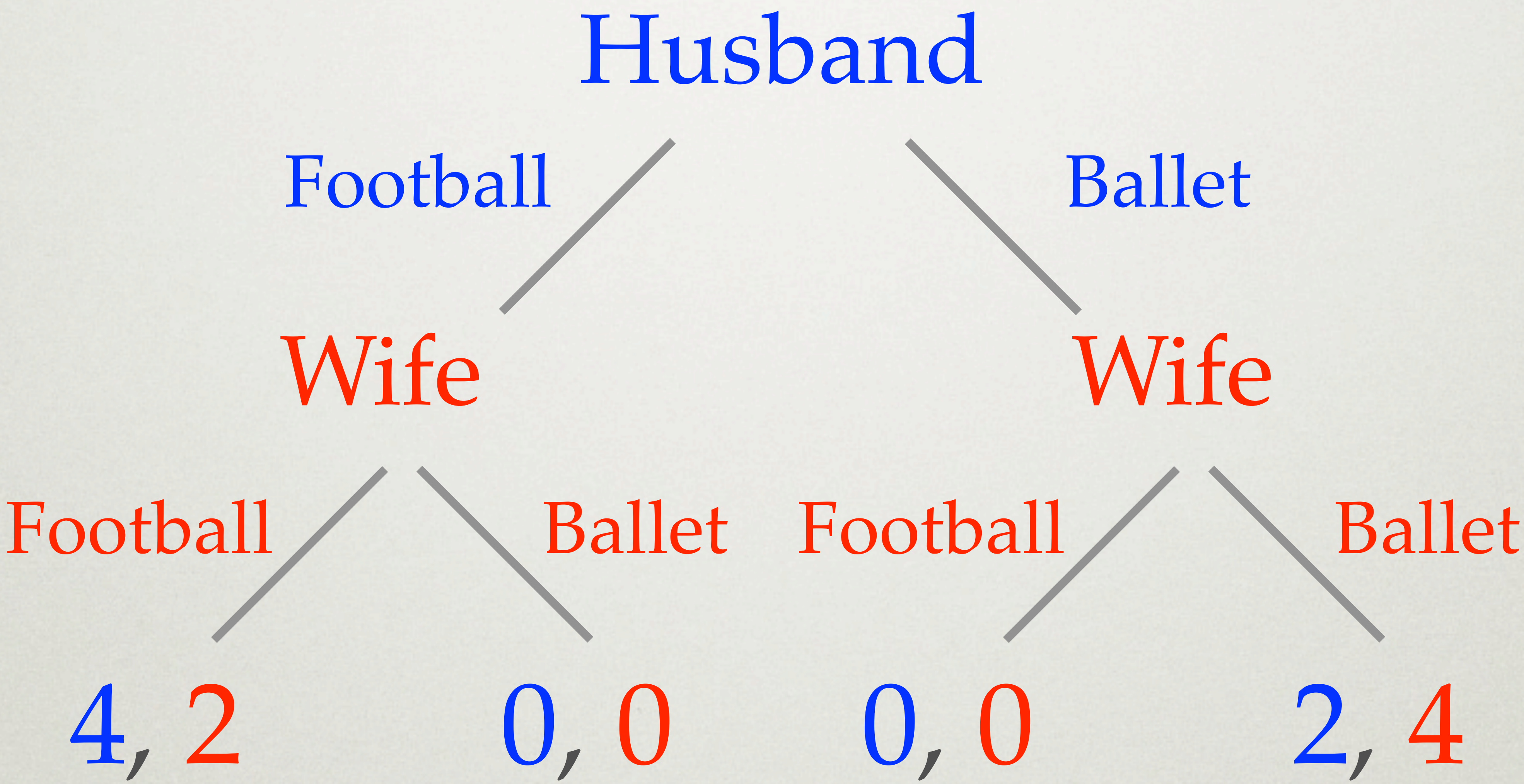
Wife

Husband	Football	Ballet
Football	4, 2	0, 0
Ballet	0, 0	2, 4

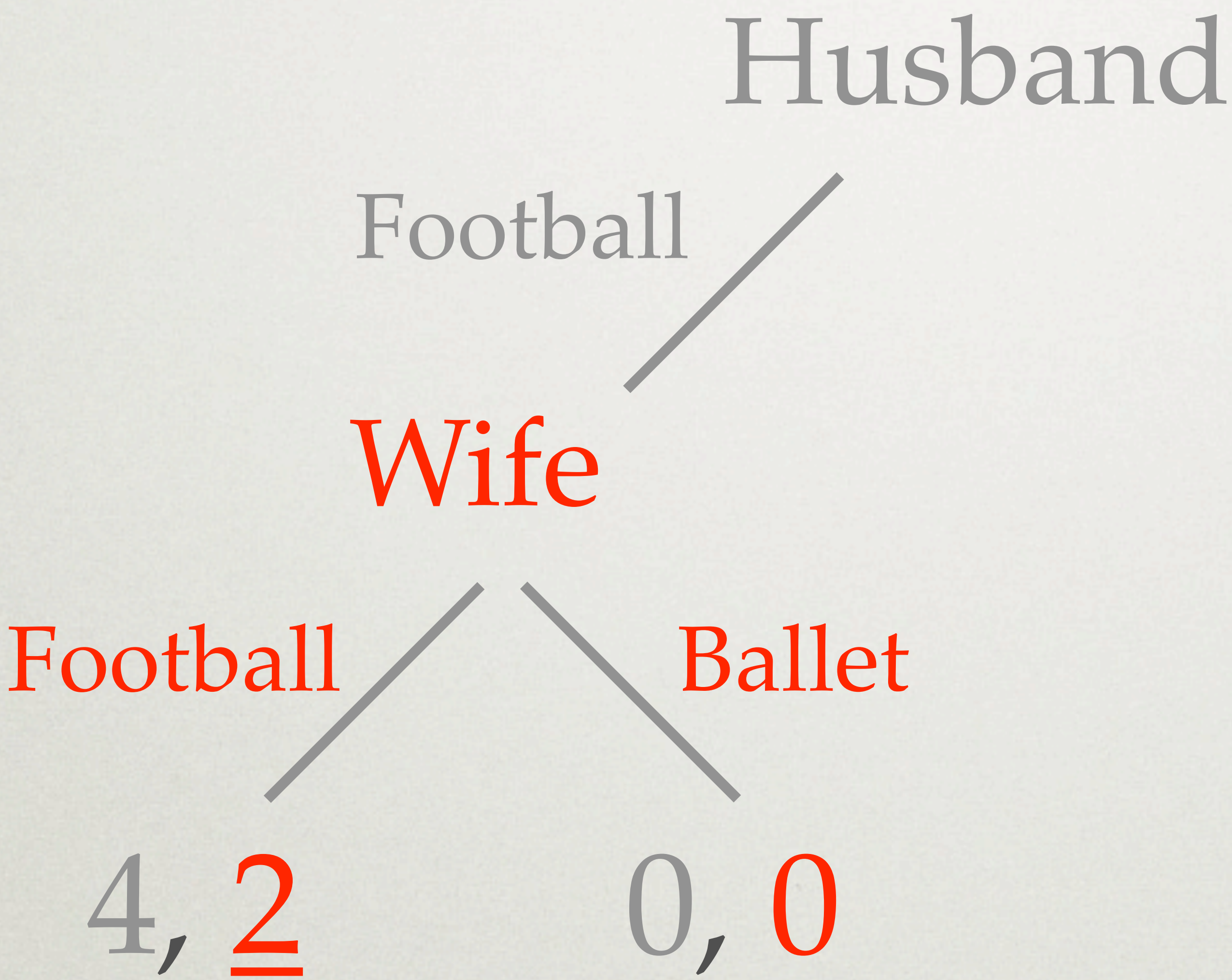
Battle of Sexes

		Wife	
		Football	Ballet
Husband	Football	$\underline{4}$, $\underline{2}$	0, 0
	Ballet	0, 0	$\underline{2}$, $\underline{4}$

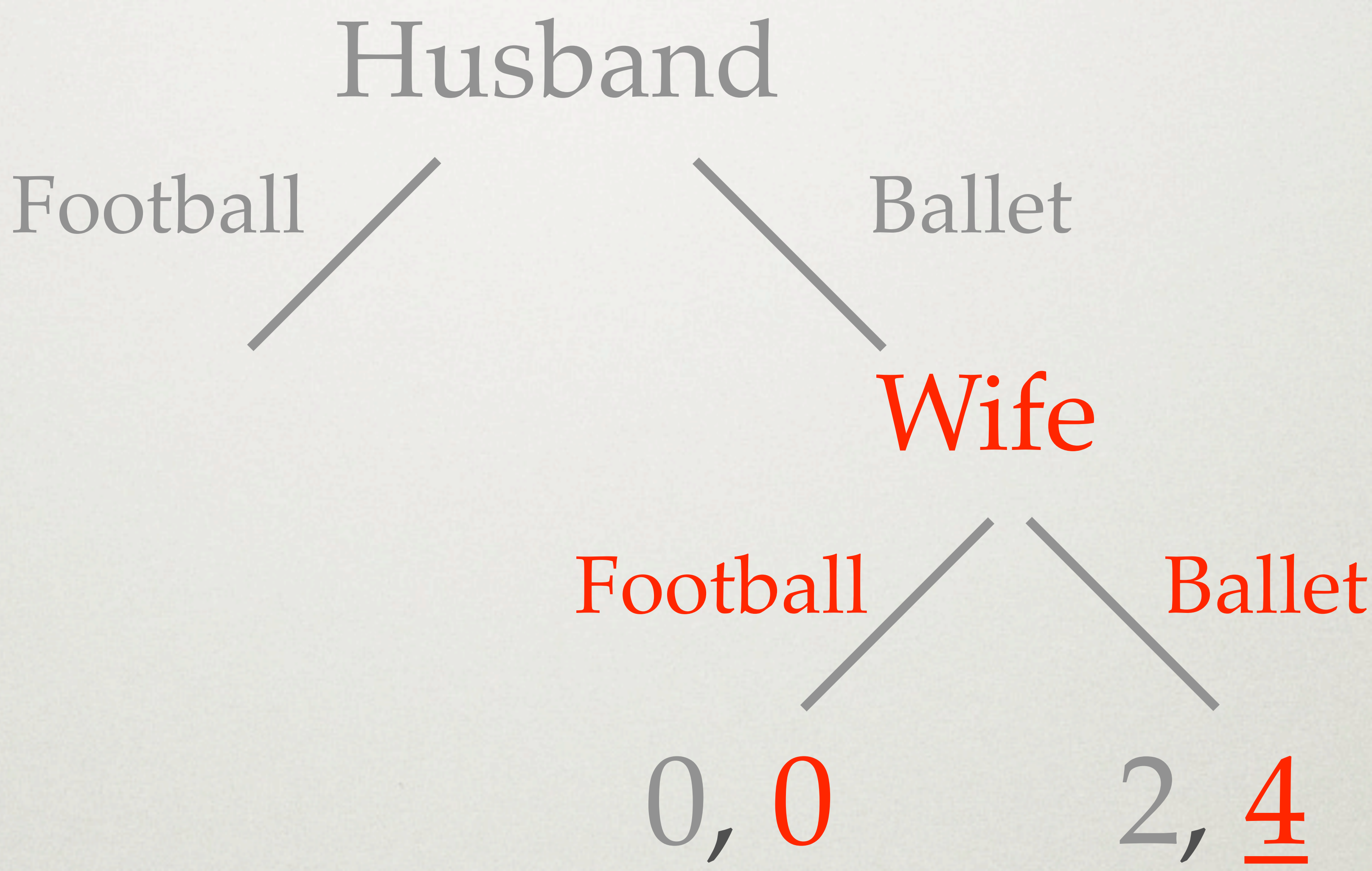
Sequential Battle of Sexes



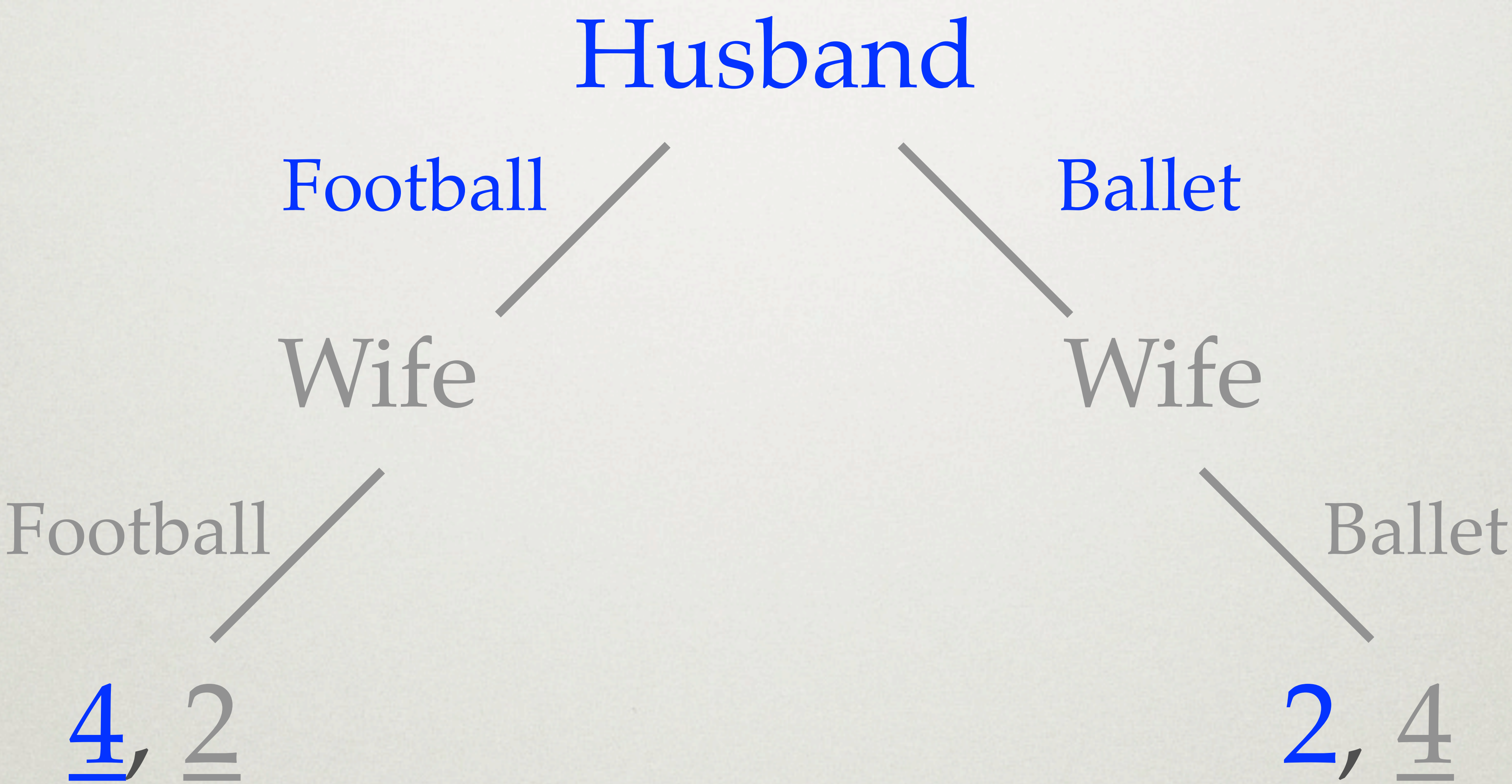
Backward Induction



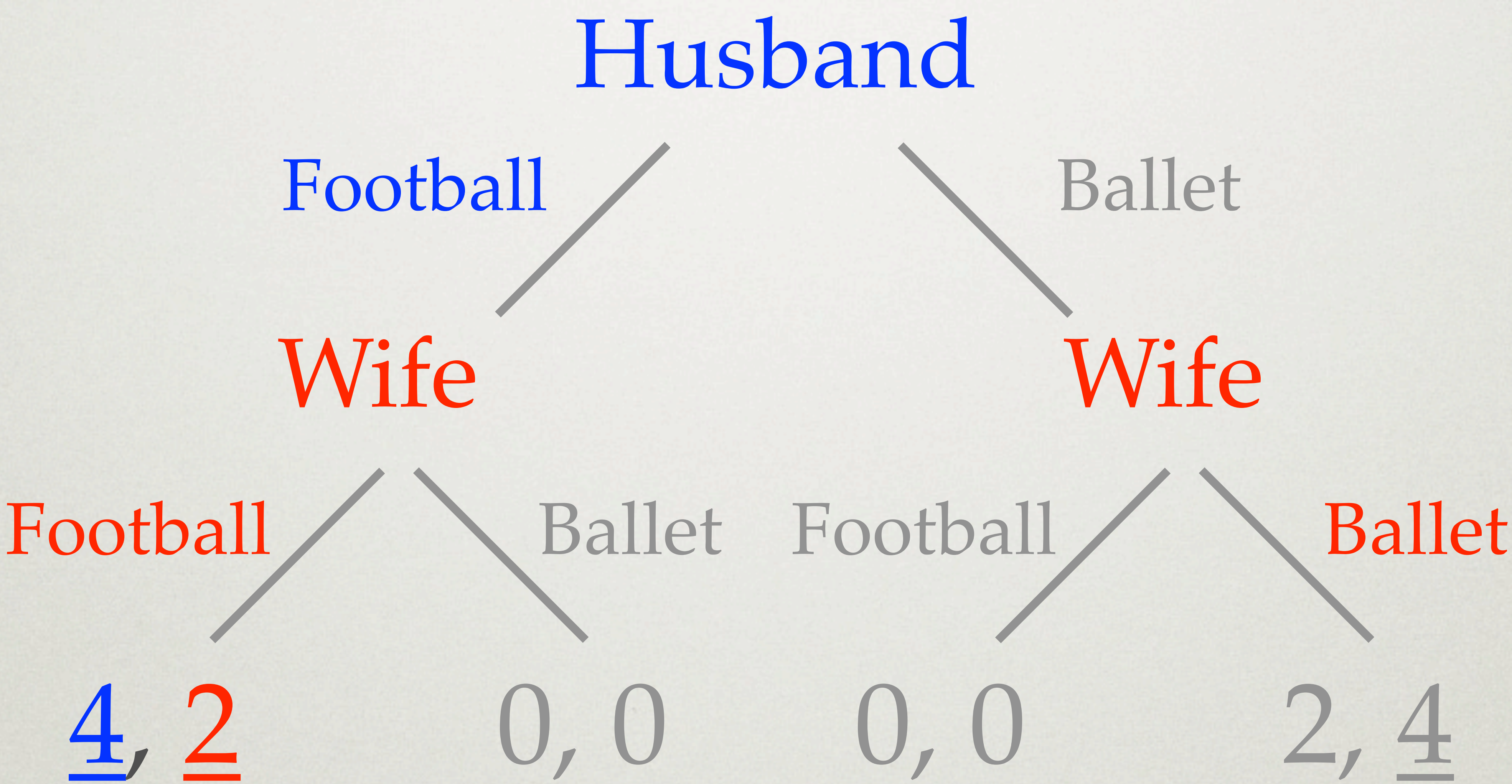
Backward Induction



Sequential Battle of Sexes



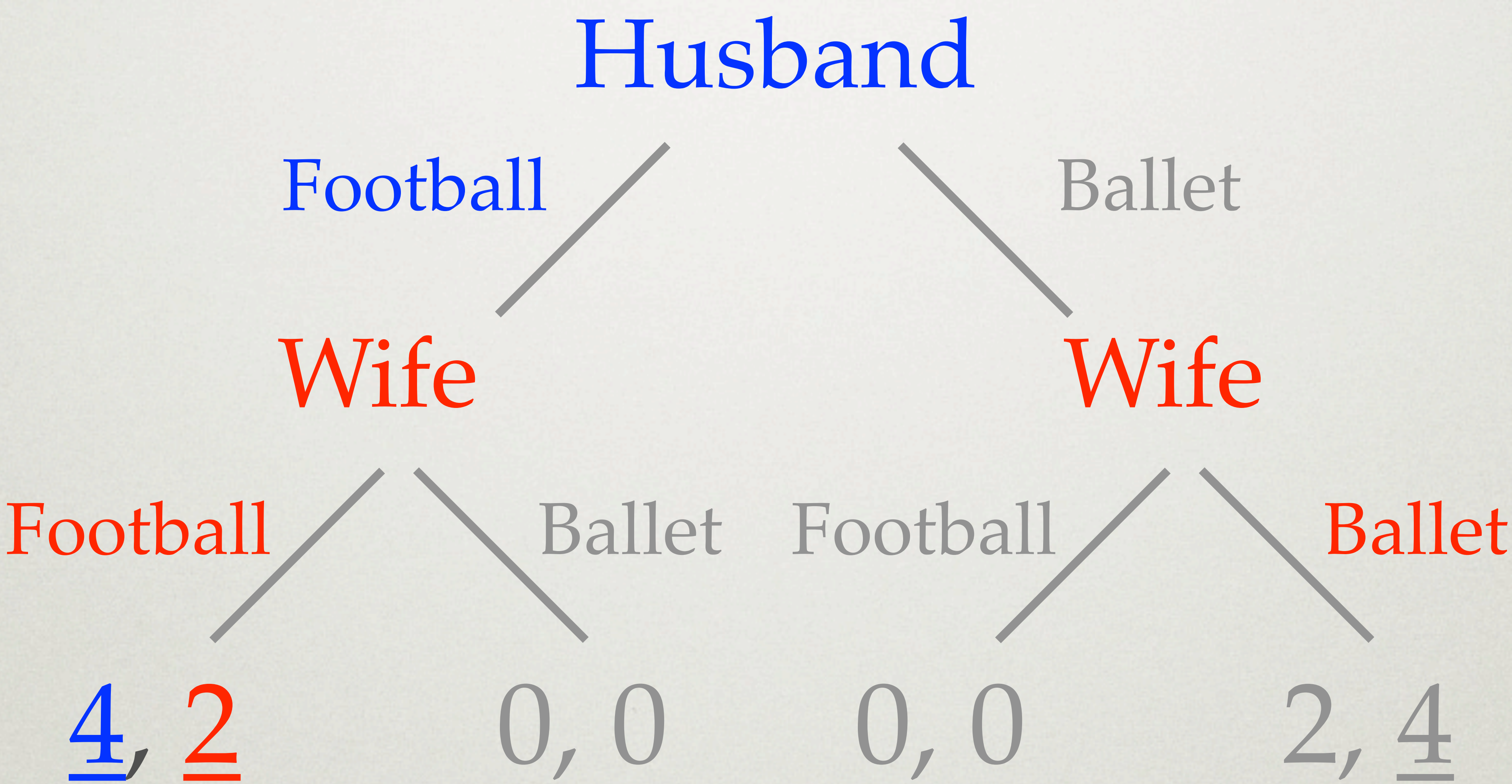
Sequential Battle of Sexes



Sequential Battle of Sexes

- Unique subgame perfect Nash equilibrium is (Football, (Football if Husband chooses Football, Ballet if Husband chooses Ballet)).
- Although the equilibrium path will be Husband picking Football and Wife picking Football, we need to specify how the Wife will pick **if the Husband picks Ballet.**
- SPNE is a **contingency plan** that specifies the action at every point in the game tree.

Sequential Battle of Sexes



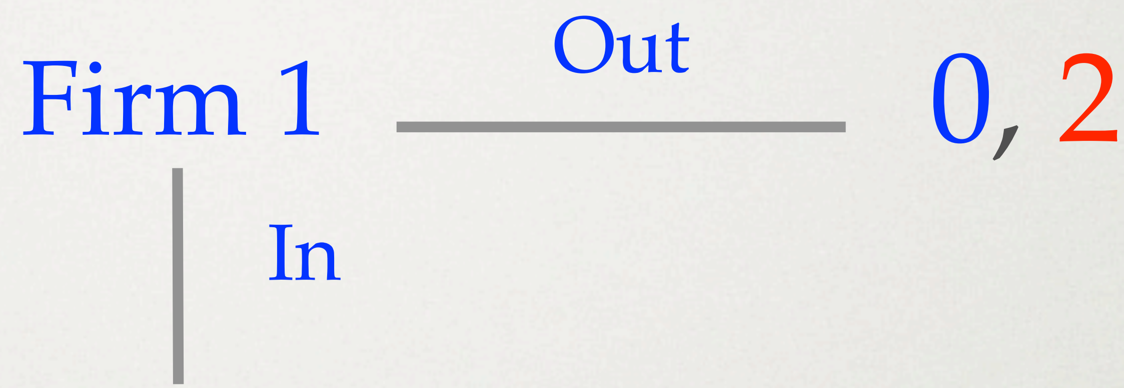
Simultaneous Moves

- Multiple players can move in the same stage.

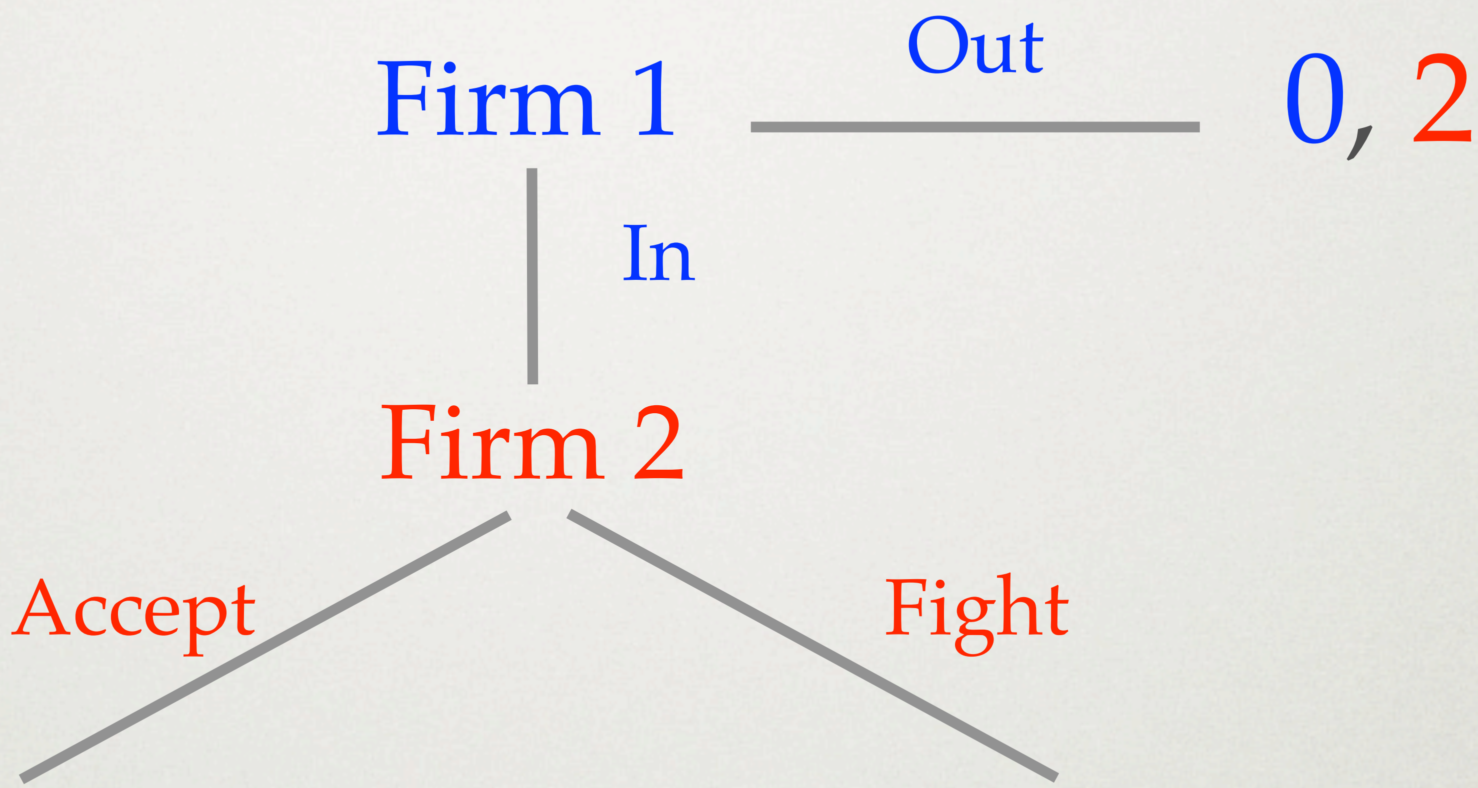
Market Entry II

- Firm 1 can choose to stay out or enter the market.
- After firm 1 enters the market, both firms need to make “accept” or “fight” decisions simultaneously, with four different possible outcomes.

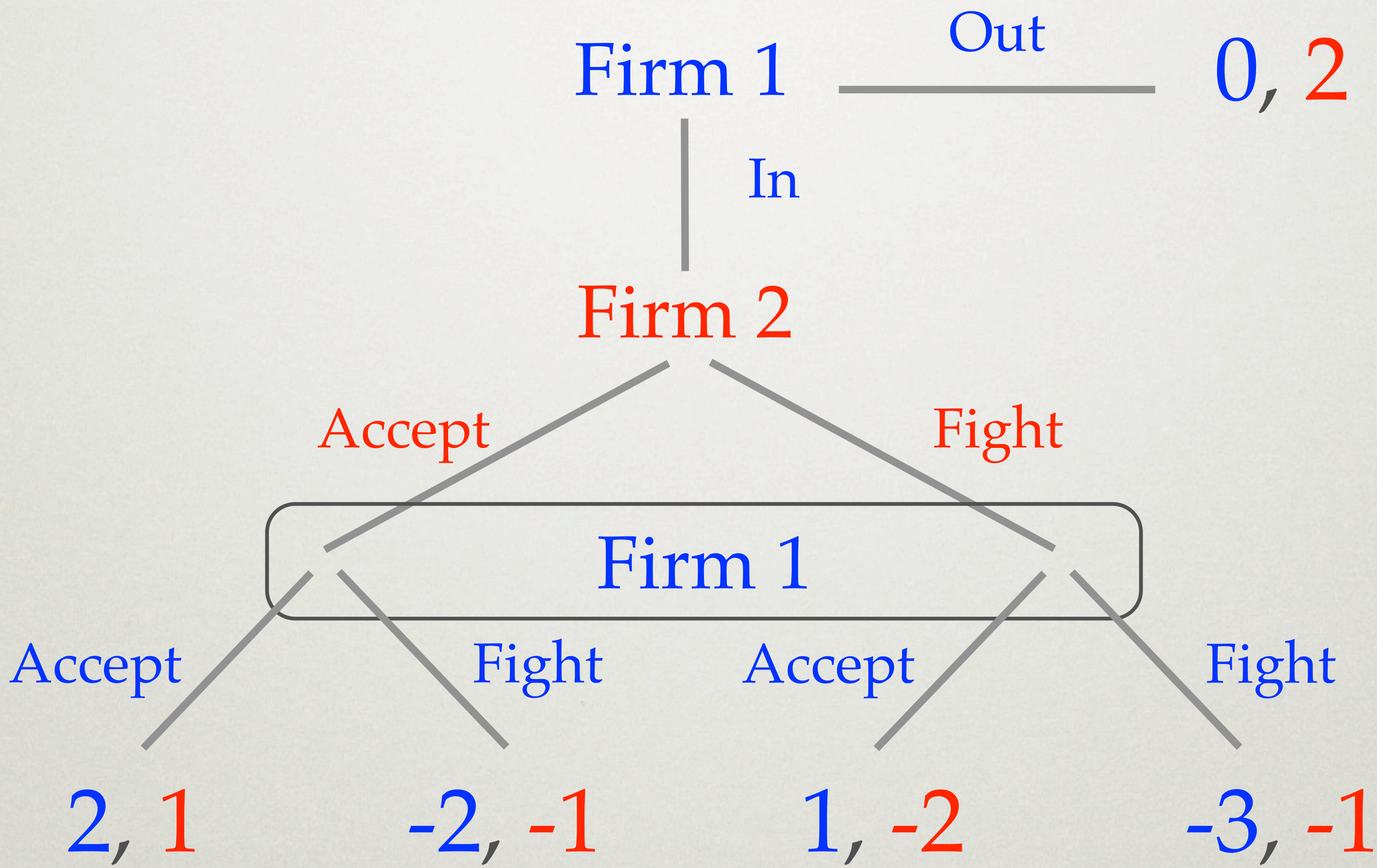
Market Entry II



Market Entry II



Market Entry II



Backward Induction

- First consider the simultaneous interactions in the second stage (after entry occurs).

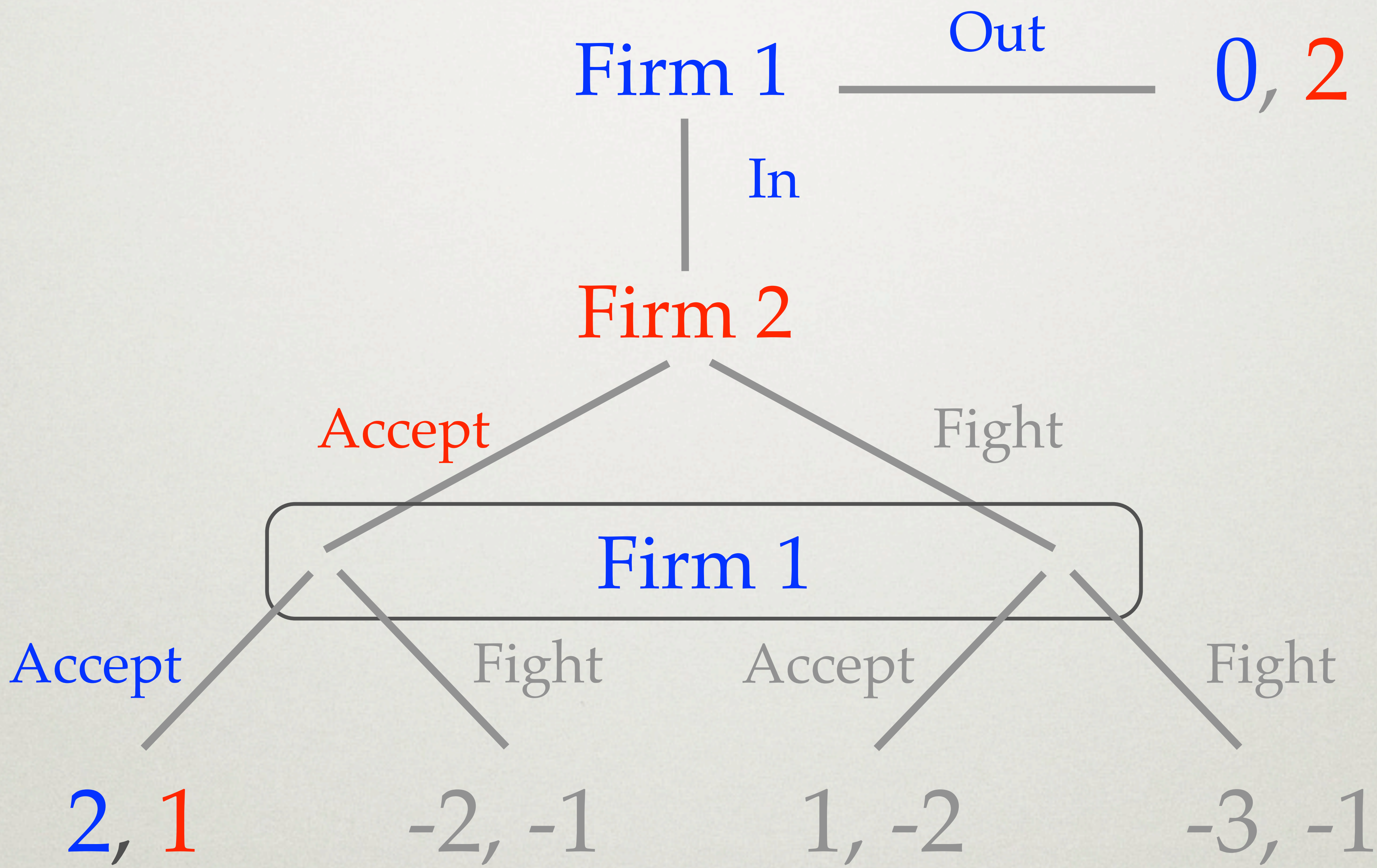
		Firm 2	
		Accept	Fight
Firm 1	Accept	2, 1	1, -2
	Fight	-2, -1	-3, -1

Backward Induction

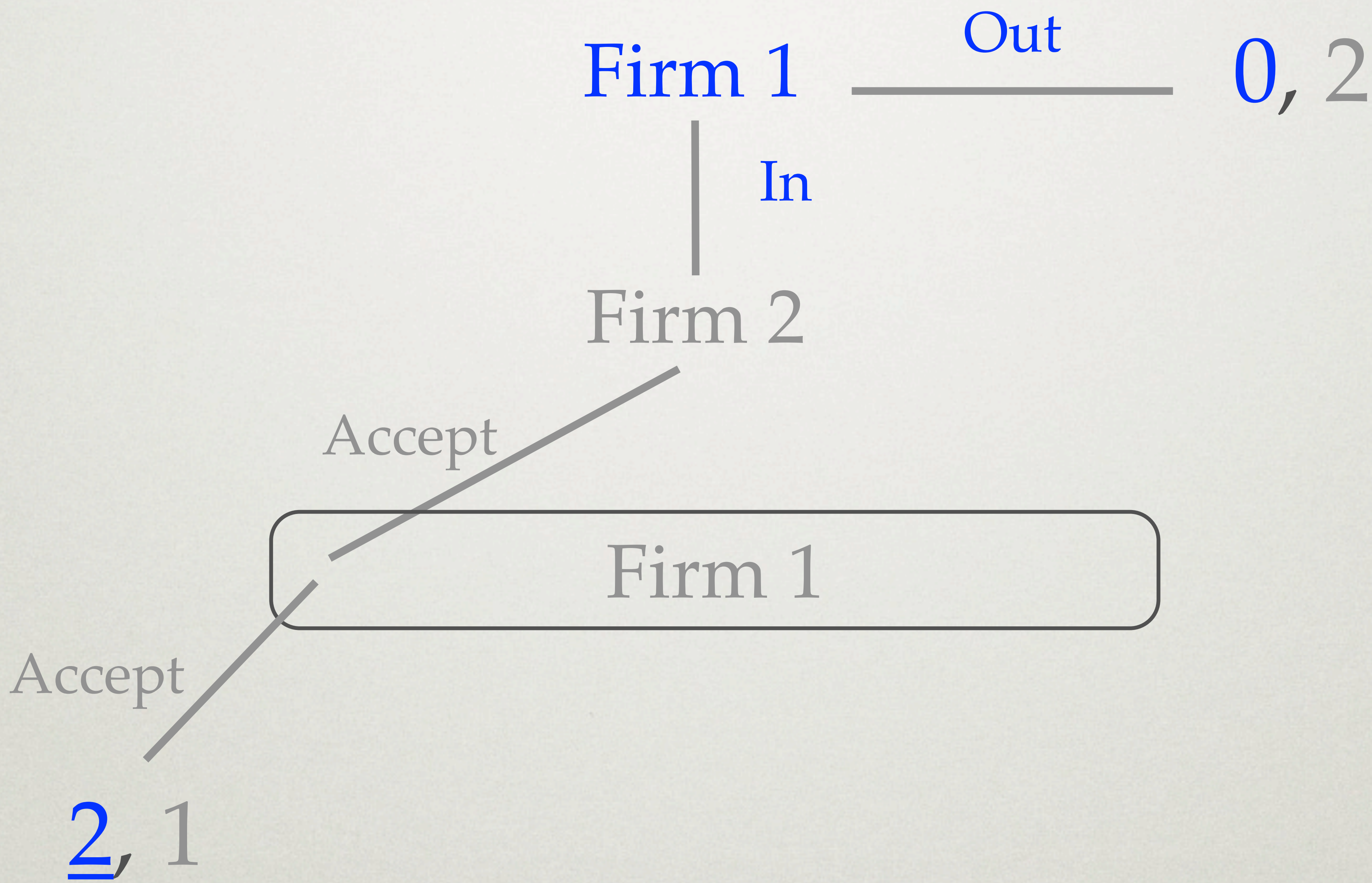
- Accept is a strictly dominant strategy for Firm 1.
- Unique Nash equilibrium is (Accept, Accept).

		Firm 2	
		Accept	Fight
Firm 1	Accept	2, 1	1, -2
	Fight	-2, -1	-3, -1

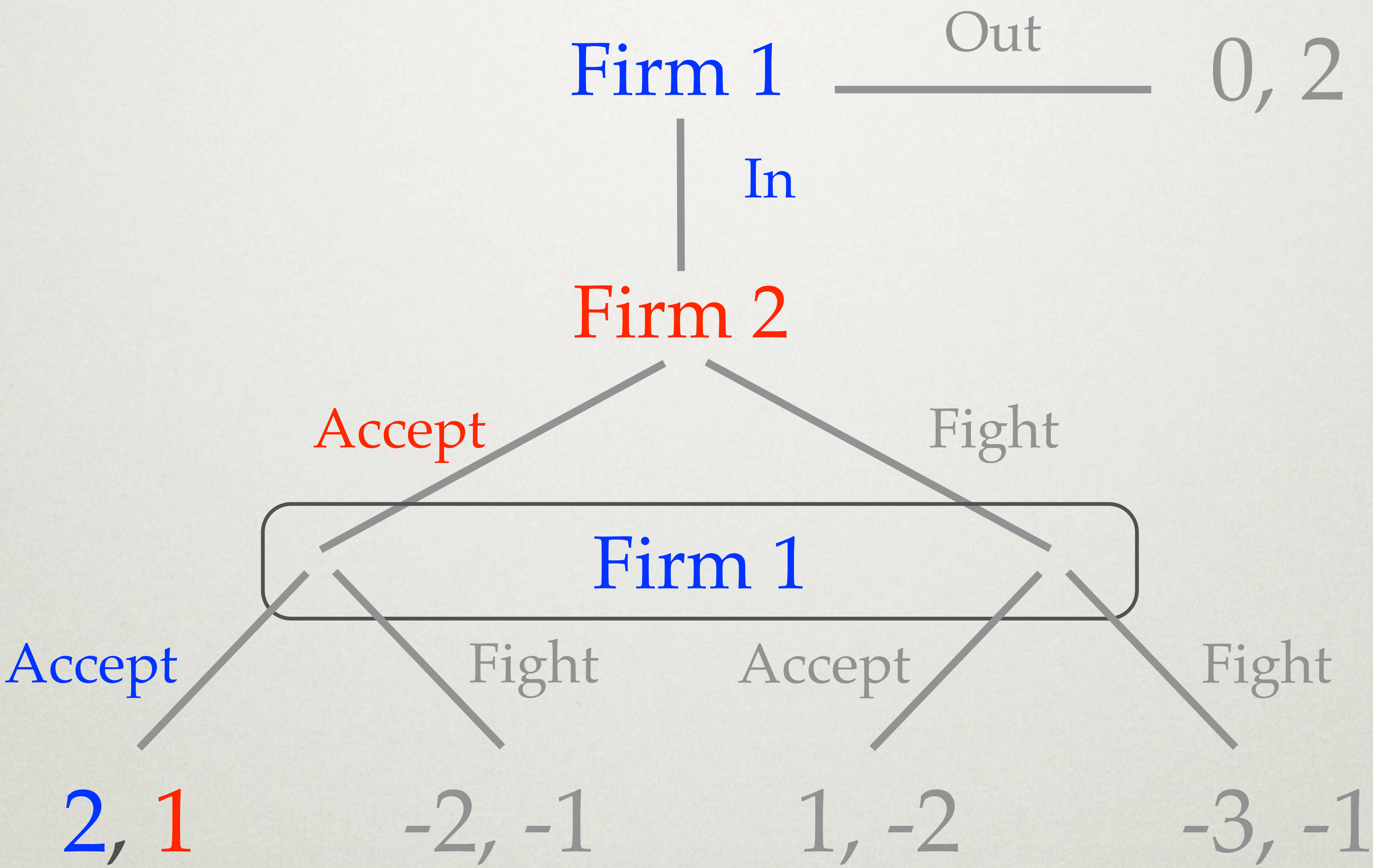
Market Entry II



Market Entry II



Market Entry II



Market Entry II

- Unique subgame perfect Nash equilibrium is ((In, Accept if entry occurs), Accept if entry occurs).

Key Concepts Review

- Subgame Perfect Equilibrium
- How to make credible threats
- Simultaneous moves in a single stage

Theory Outline

- Game theory:
  - Static games
  - Dynamic games
- Economics:
  - Price discrimination
  - Network Externality

Economics:
Price Discrimination

Price Discrimination

- A company sells one type of product to consumers to maximize profit (revenue minus cost).
- No price discrimination: charge the same price for
  - Each consumer
  - Each unit of the product
- Price discrimination: changing one or two of the above assumptions

Three Types

- First-degree: Perfect price discrimination
  - Charge each consumer the most he is willing to pay for each unit of product.
- Second-degree: Declining block pricing
  - Charge different prices for different units of products, but not differentiating consumers.
- Third-degree: Multi-market price discrimination
  - Charge different prices for different consumers, but not differentiating products.

Example

- A single product with no cost.
- Alice is wiling to pay \$10 for the 1st unit and \$2 for the 2nd unit. Bob is willing to pay \$7 for a single unit.
- Maximum revenue w/o differentiation: \$14.

- First-degree: charge Alice \$10+\$2 for two units, and Bob \$7 for one unit. Revenue: \$19.
- Second-degree: charge \$7 for one unit, and \$12 for two units. No consumer difference. Revenue: \$19.
- Third: charge Alice \$6 per unit, and Bob \$7 per unit. No quantity discount. Revenue: \$19.

How to Discriminate

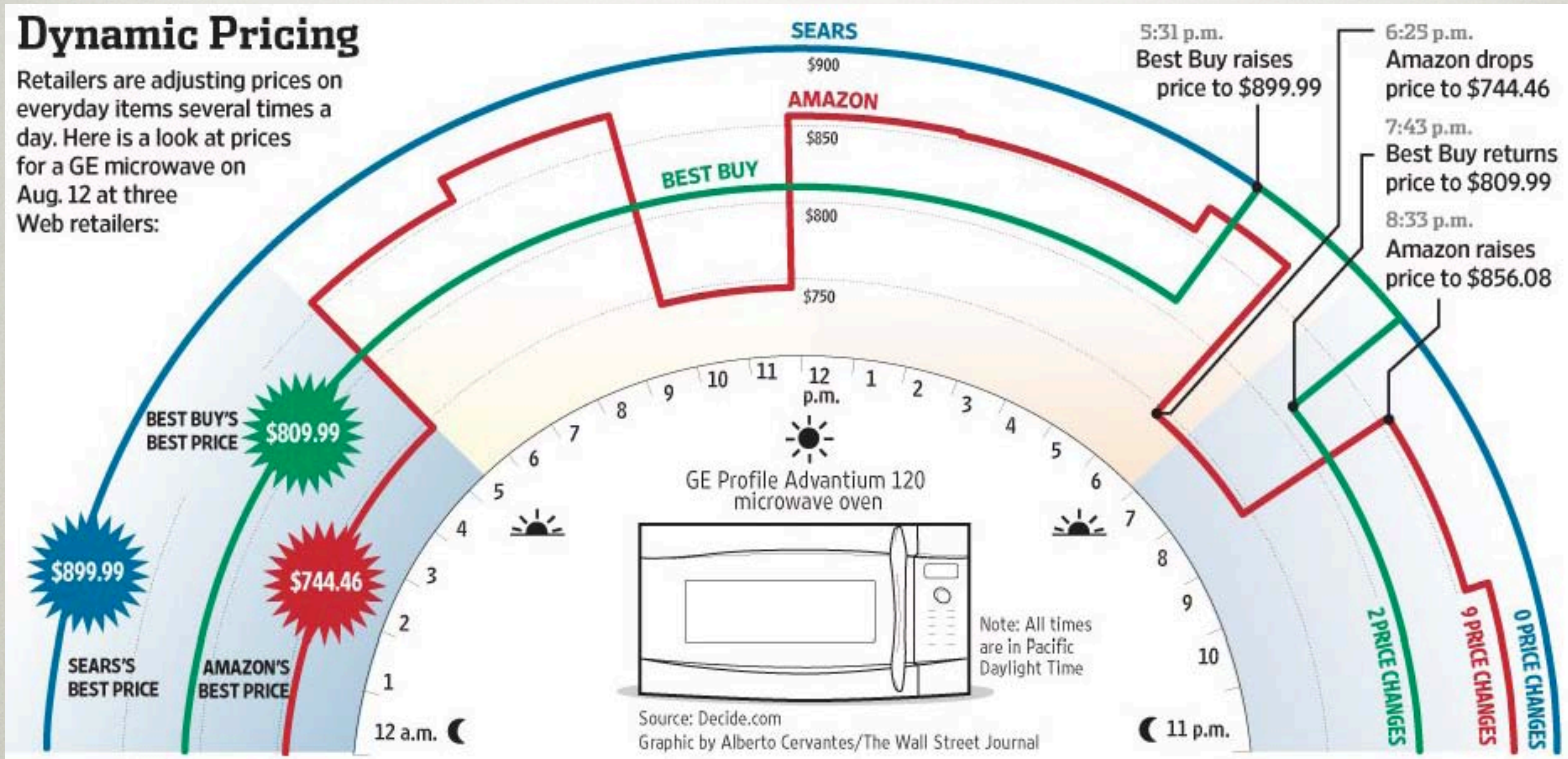
- Identify consumer types
  - Age
  - Time
  - ...
- Prevent resale
  - Using photo ID for airline tickets
  - ...

By Age

By Time

- Kindle 1
  - 11/07, $399
- Kindle 2
  - 2/09, $399; 7/09, $299
  - 10/09, $259; 6/10, $189
- Kindle 3,
  - 8/10, $139
  - 9/11, $79

Even more Dynamic



More innovative Ones

- Orbitz shows more expensive hotel options to Mac users than windows users (source: WSJ 08/12)

Economics:
Network Externality

Network Externality

- Any side effect imposed by the action of a player on a third party not directly involved.
- Can be either negative (cost) or positive (benefits).

Negative Externality

Negative Externality

Negative Externality

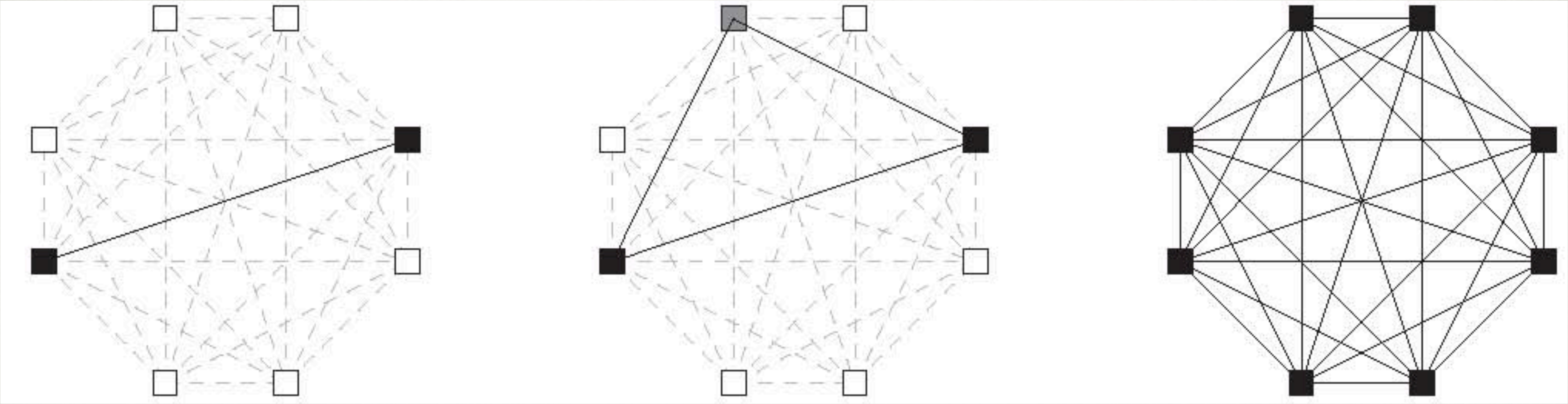
Negative Externality

- Negative externality distorts the market and reduces social welfare
- How to correct: Pigovian tax (one approach)
  - Impose additional tax on entities generating the negative externalities
  - Examples: pollution tax, cigarette taxes ($1.01 per pack of US federal tax in 2009) , congestion pricing (Electronic Road Pricing in Singapore)

Positive Externality

Positive Externality

Network Effect



Network Effect

- Metcalfe's law'80
  - A network with n nodes has up to n(n-1)/2 unique connections
  - Hence the network value is roughly O(n^2)
- Briscoe-Odlyzko-Tilly'06 refinement
  - Not all connections are equally important
  - The importance of connections decreases as 1, 1/2, 1/3, ..., 1/(n-1), with the sum ~ log(n)
  - A network value grows O(n*log(n))

Theory Outline

- Game theory:
  - Static games
  - Dynamic games
- Economics:
  - Price discrimination
  - Network Externality

Applications

- Graphical congestion games (static game)
- Spectrum sensing-leasing tradeoff (dynamic games)
- Spectrum leasing competition (oligopoly competition)
- Partial price differentiation (price differentiation)
- Distributed power control (negative network externality)
- Cellular network upgrade (positive network externality)

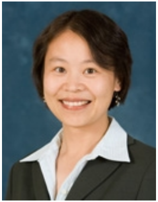
Our Focus

- Key motivation
- Key modeling
- Key methodology
- More results can be found in the papers

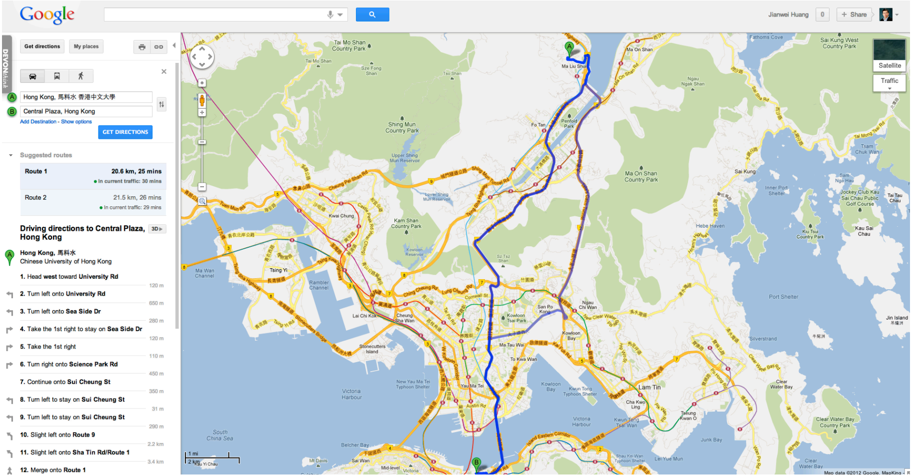
Graphical Congestion Games

- R. Southwell, X. Chen, and J. Huang, "Quality of Service Games for Spectrum Sharing," *IEEE Journal on Selected Areas of Communications*, 2014
- X. Chen and J. Huang, "Distributed Spectrum Access with Spatial Reuse," *IEEE Journal on Selected Areas in Communications*, 2013
- C. Tekin, M. Liu, R. Southwell, J. Huang, and S. Ahmad, "Atomic Congestion Games on Graphs and Their Applications in Networking," *IEEE/ACM Transactions on Networking*, 2012



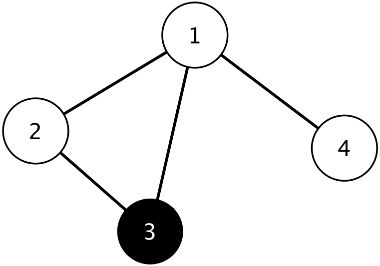



Congestion Game



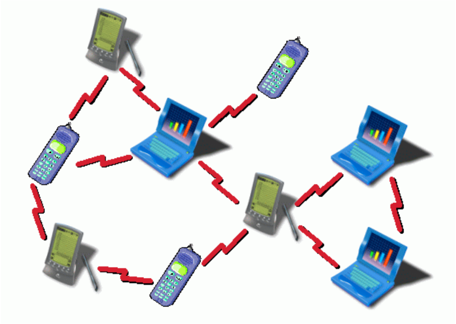
- Each user chooses which resource to use considering congestion

Graphical Congestion Game



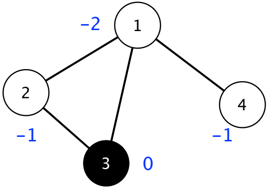
- Graph characterizes users' relationship
  - Nodes: users
  - Edges: potential congestion relationship
  - Colors: resource choices
- Users 2 and 4 will never generate congestion to each other

Channel Selection and Interference Management



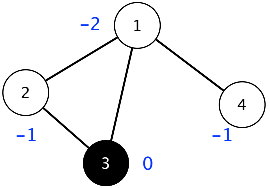
- Users: mobile devices
- Resources: channels
- Congestion: interferences

Graphical Congestion Games (GCG) Model



- Players (users): $\mathcal{N} = \{1, 2, 3, 4\}$

Graphical Congestion Games (GCG) Model



- Players (users): $\mathcal{N} = \{1, 2, 3, 4\}$
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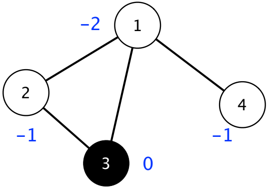

- Players (users): $\mathcal{N} = \{1, 2, 3, 4\}$
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- Graph: $S = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$

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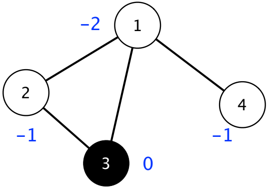

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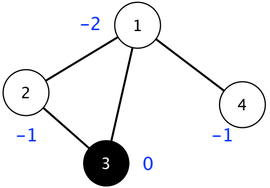

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- Payoff: $f_n^r(\mathbf{X}) = f_n^r(\sum_{X_m=r} S_{m,n}),\ \forall r \in \mathcal{R}, \forall n \in \mathcal{N},$

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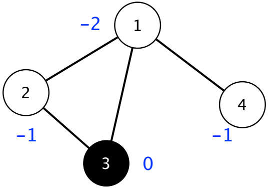

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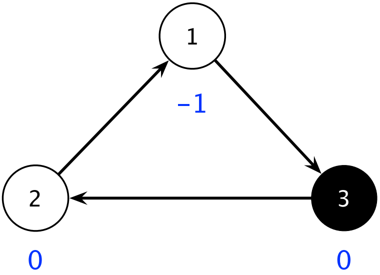
- Payoff: $f_n^r(\mathbf{X}) = f_n^r(\sum_{X_m=r} S_{m,n}), \ \forall r \in \mathcal{R}, \forall n \in \mathcal{N},$
- In general, the graph can be weighted and directed

Key Question 1

- Does GCG have a unique Pure Nash equilibrium (PNE)?

PNE May Not Exist



- No PNE: at least one player can improve its payoff by switching.

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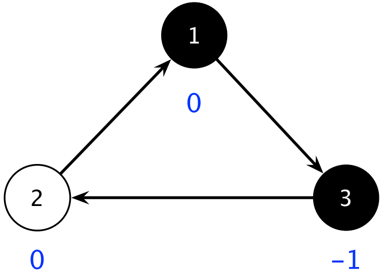

- No PNE: at least one player can improve its payoff by switching.
- Player 1 switches, but player 3 becomes unsatisfied.

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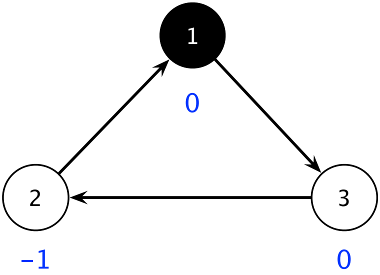
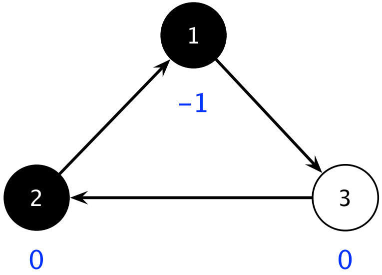


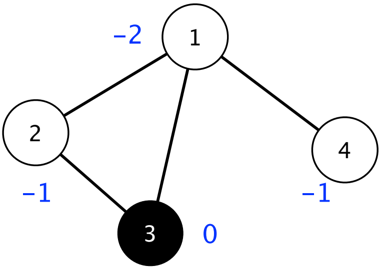
- No PNE: at least one player can improve its payoff by switching.
- Player 1 switches, but player 3 becomes unsatisfied.
- Player 3 switches, but player 2 becomes unsatisfied.

PNE May Not Exist



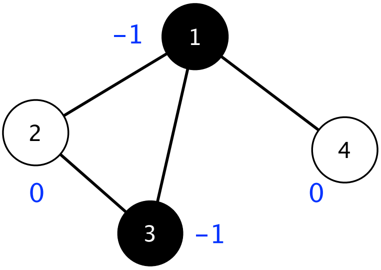
- No PNE: at least one player can improve its payoff by switching.
- Player 1 switches, but player 3 becomes unsatisfied.
- Player 3 switches, but player 2 becomes unsatisfied.
- Player 2 switches, but player 1 becomes unsatisfied.

PNE May Not Be Unique



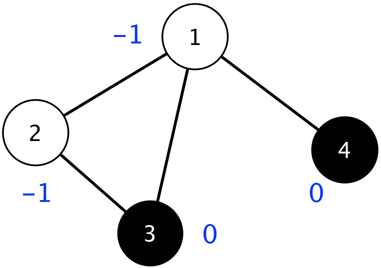
- This is not a PNE: player 1 can improve by switching to Black.

PNE May Not Be Unique



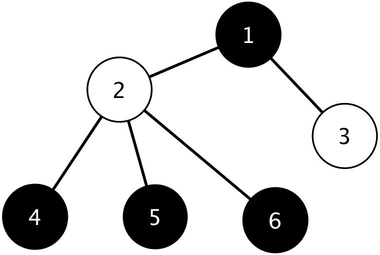
- This is not a PNE: player 1 can improve by switching to Black.
- This is a PNE.

PNE May Not Be Unique

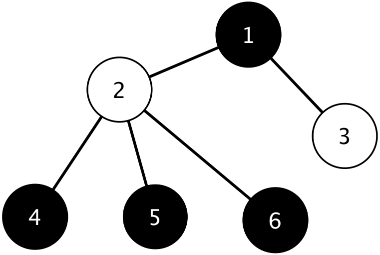


- This is not a PNE: player 1 can improve by switching to Black.
- This is a PNE.
- This is another PNE.

Existence of PNE: Tree

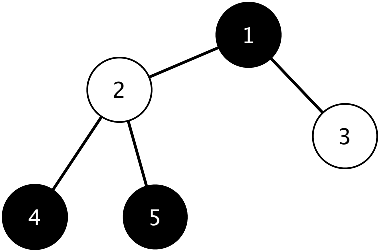


Existence of PNE: Tree



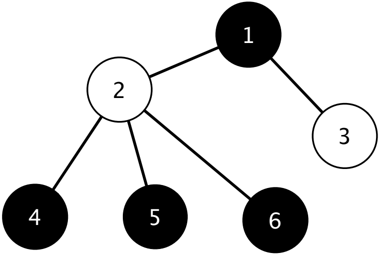
- Proof idea:

Existence of PNE: Tree



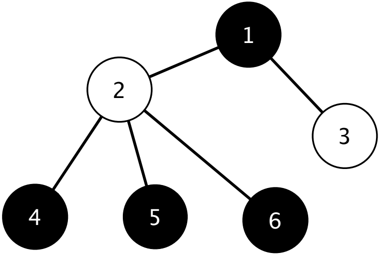
- Proof idea:
  - Consider a GCG with a PNE.

Existence of PNE: Tree



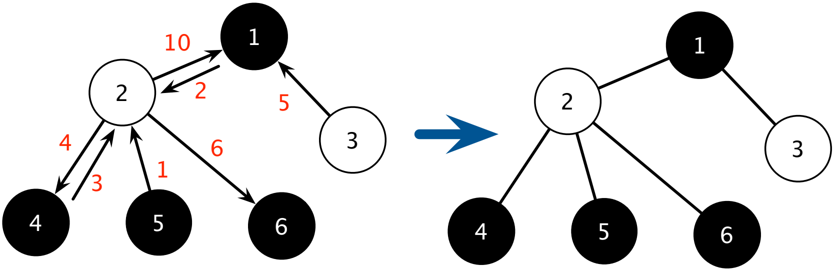
- Proof idea:
  - Consider a GCG with a PNE.
  - Add a new player with a single connection with the original GCG.

Existence of PNE: Tree



- Proof idea:
  - Consider a GCG with a PNE.
  - Add a new player with a single connection with the original GCG.
  - Show that the new GCG also has a PNE.

Existence of PNE: Directed Weighted Tree

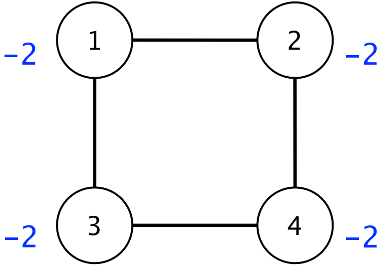


- Directed weighted tree: the corresponding undirected graph is a tree.

Key Question 2

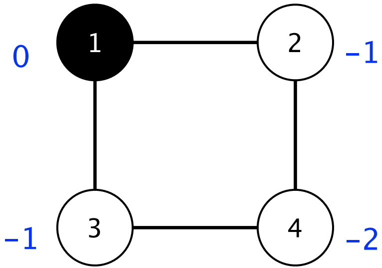
- How to achieve a PNE?

Asynchronous Better Response



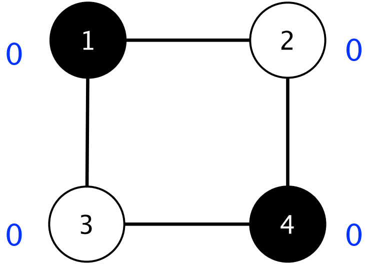
- Asynchronous better response updates: players improve, one at a time

Asynchronous Better Response



- Asynchronous better response updates: players improve, one at a time
- Step 1: Player 1 switches to Black

Asynchronous Better Response



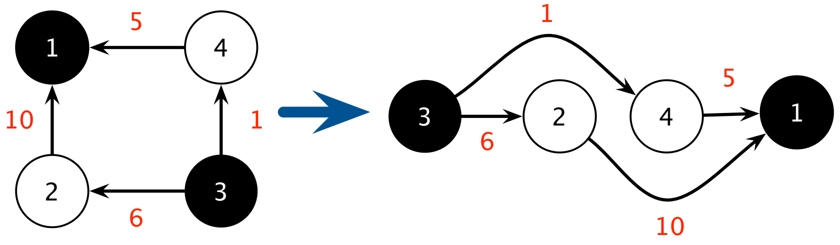
- Asynchronous better response updates: players improve, one at a time
- Step 1: Player 1 switches to Black
- Step 2: Player 4 switches to Black, and reaches a PNE

Finite Improvement Property (FIP)

Definition (FIP)

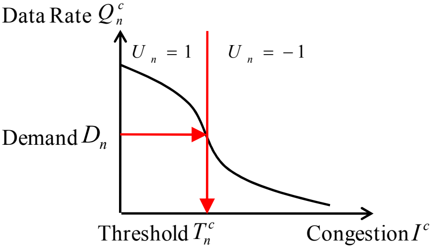
A GCG has the Finite Improvement Property (FIP) if every sufficiently long sequence of better response updates leads to a PNE.

Existence of FIP: Directed Acyclic Graph



- Directed acyclic graph: graph does not contain cycles.
- Existence of PNE:
  - Create a topological sort: $3, 2, 4, 1$
  - Construct a PNE by letting players sequentially update their strategies
- Can further prove the existence of FIP.

Case Study: Spatial QoS Satisfaction Games



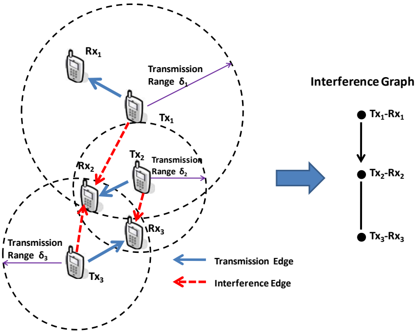
- Spatial QoS satisfaction game always has the FIP.
- With homogenous users: any PNE is socially optimal.
- With homogeneous channels: design an algorithm to generate a socially optimal PNE.

Modeling Wireless Channel Selections

- Protocol interference model
- Undirected unweighted graph: symmetric interference relationship

Modeling Wireless Channel Selections

- Protocol interference model
- Undirected unweighted graph: symmetric interference relationship
- Directed unweighted graph: users have different transmission/interference ranges



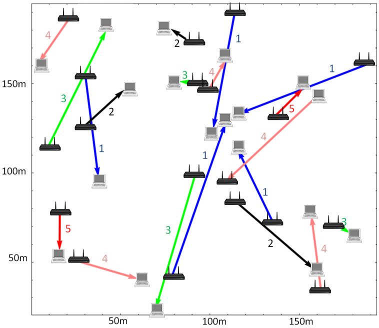
Modeling Wireless Channel Selections

- Physical interference model
- Data rate increasing in signal-to-interference-plus-noise ratio (SINR)

$$\mathtt{SINR} = \frac{h_{n,n}P_n}{\tau_0 B_i + \sum_{m:m\neq n, X_m=r} h_{m,n}P_m}.$$

- Interference is weighted and asymmetric: $\sum_{m:m\neq n, X_m=r} h_{m,n}P_m$
- Need to consider directed and weighted graph

Simulation Setup



- Users are uniformly distributed in an area with size $L \times L\ m^2$.
- Fixed user transmission power $P_n = 100$mW.
- Channel bandwidth of $B_r = 20$MHz.
- User payoff equals data rate $\log(1 + \mathtt{SINR})$.
- Distance-based channel gain $h_{m,n} = 1/d_{m,n}^4$.

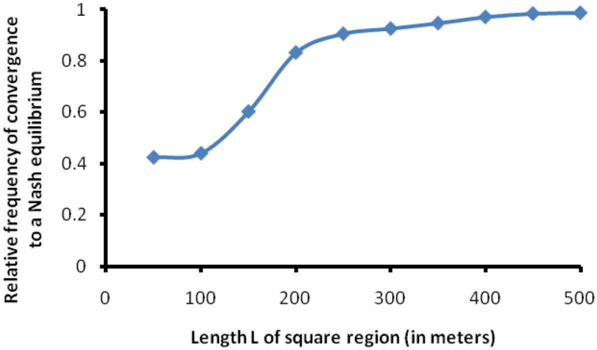
Properties of Graphs

- The underlying graph is weighed, directed, with loops.
  - A PNE may not exist.

Properties of Graphs

- The underlying graph is weighed, directed, with loops.
  - A PNE may not exist.
- As network size L increases, interferences become approximately symmetric
  - Users can be approximated as dots in the network
  - The graph becomes undirected and weighted
  - Theory implies that GCG has FIP, and thus a PNE exists.

Percentage of Convergence



- Count convergence faster than 500 slots.

Generalization of Payoff Functions

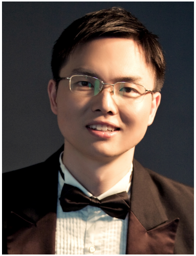
- Modeling more general (wireless) resource sharing mechanisms
- Example: payers share channels based on *p*-persistent random access with player-specific contending probability

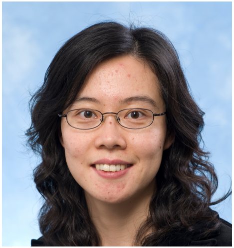
$$U_n(\mathbf{X}) = \theta_{X_n} B^n_{X_n} g_n(\mathcal{N}^{X_n}_n(\mathbf{X})) = \theta_{X_n} B^n_{X_n} p_n \prod_{i \in \mathcal{N}^{X_n}_n(\mathbf{X})} (1 - p_i)$$

- Construct special potential function to prove FIP.

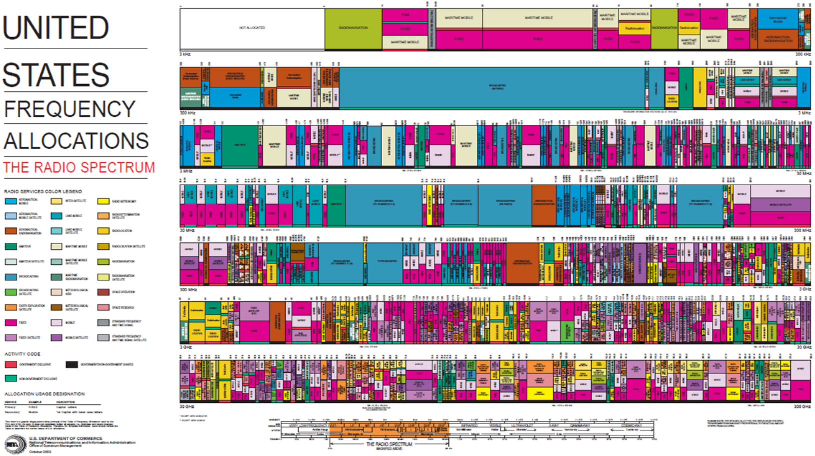
Spectrum Sensing-Leasing Tradeoff

L. Duan, J. Huang, and B. Shou, "Investment and Pricing with Spectrum Uncertainty: A Cognitive Operators Perspective," *IEEE Transactions on Mobile Computing*, 2011

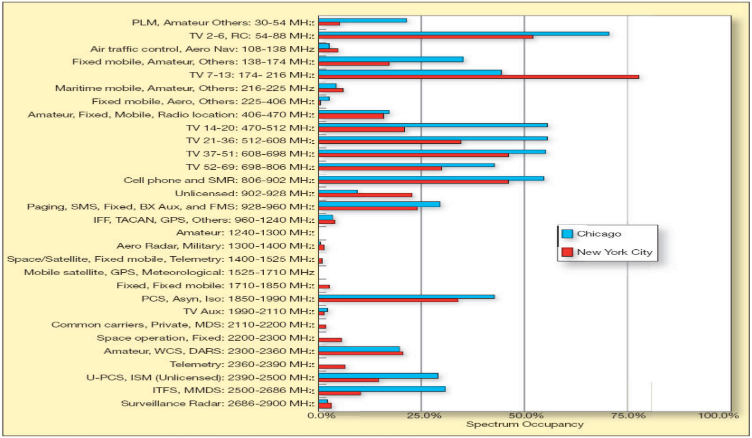




Spectrum Is Scarce



Spectrum Is Under-Utilized



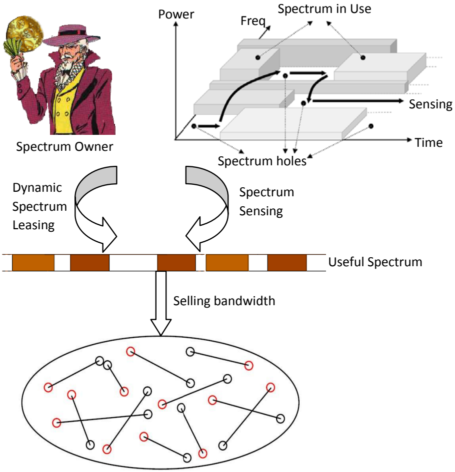
Cognitive Virtual Network Operators

- Virtual: does not own radio spectrum (or even physical infrastructure)
- Flexible spectrum acquisition

Investment Choices	Dynamic Leasing	Spectrum Sensing
Cost	High	Low
Reliability	High	Low

- Pricing & spectrum allocation among local users to maximize profit

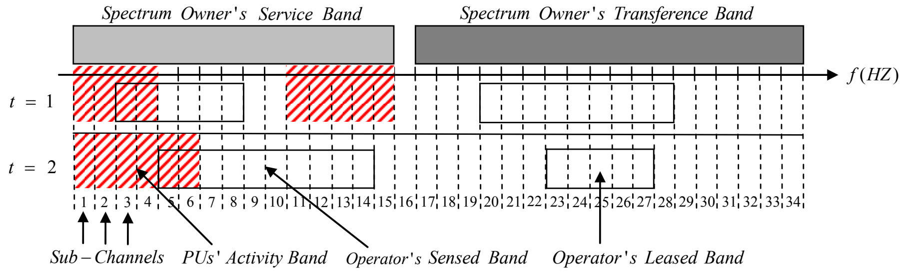
Network Model



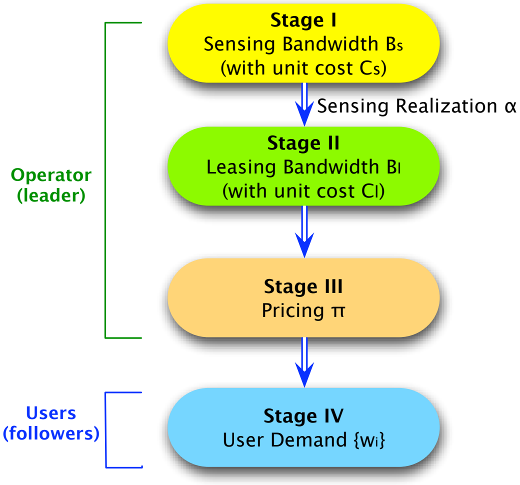
Users (transmitter-receiver node pairs)

Two Spectrum Investment Choices

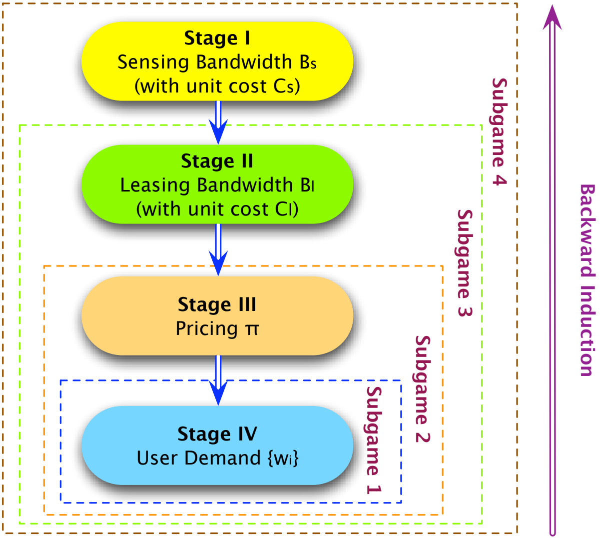
- Both on a short time scale



Four-Stage Stackelberg Game



Backward Induction & Subgame Perfect Equilibrium



Stage IV: Users' Bandwidth Demands

- Physical layer model: users share the spectrum using OFDM
  - No interferences
  - Users request bandwidth from the operator
- User k's wireless characteristics:

$$g_k = \frac{P_k^{\max} h_k}{n_0}$$

  - $P_k^{\max}$: maximum transmission power
  - h_k: channel condition
  - n_0: background noise density
- User k's data rate

$$r_k(w_k) = w_k \ln(1 + \mathtt{SNR}_k) = w_k \ln\left(1 + \frac{g_k}{w_k}\right)$$

Users' Payoff Functions

- Assume that all users operate in the high SNR regime

$$r_k(w_k) \approx w_k \ln\left(\frac{g_k}{w_k}\right)$$

- User k's payoff

$$u_k(\pi, w_k) = w_k \ln\left(\frac{g_k}{w_k}\right) - \pi w_k$$

Users' Optimization Problems

User i's Bandwidth Optimization Problem

$$w_k^*(\pi) = \arg\max_{w_k \geq 0} u_k(\pi, w_k) = g_k e^{-(1+\pi)}$$

Users' Optimization Problems

User i's Bandwidth Optimization Problem

$$w_k^*(\pi) = \arg\max_{w_k \geq 0} u_k(\pi, w_k) = g_k e^{-(1+\pi)}$$

- $\mathtt{SNR}_k^* = g_k / w_k^* = e^{1+\pi}$: same (fair) for all users
- Payoff $u_k(\pi, w_k^*) = g_k e^{-(1+\pi)}$: linear in g_k

Stages III, II and I

- Stage III: operator optimizes over price π:

$$R_{III}(B_l, B_s, \alpha) = \max_{\pi \geq 0} \min \left(\pi \sum_k w_k^*(\pi), \pi \left(B_l + B_s \alpha\right) \right) - (B_s C_s + B_l C_l)$$

Stages III, II and I

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- Stage II: operator optimizes over leasing bandwidth B_l:

$$R_{II}(B_s, \alpha) = \max_{B_l \geq 0} R_{III}(B_l, B_s, \alpha).$$

Stages III, II and I

- Stage III: operator optimizes over price π:

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- Stage II: operator optimizes over leasing bandwidth B_l:

$$R_{II}(B_s, \alpha) = \max_{B_l \geq 0} R_{III}(B_l, B_s, \alpha).$$

- Stage I: operator optimizes over sensing bandwidth B_s:

$$\max_{B_s \geq 0} E_{\alpha \in [0,1]} \left[R_{II}(B_s, \alpha) \right].$$

  - **Assumption**: sensing uncertainty α follows uniform distribution.
  - Will be relaxed later.

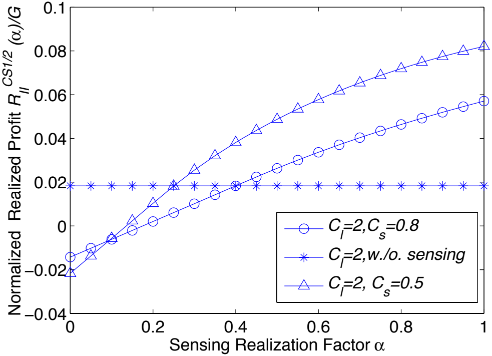
Equilibrium Summary

- Unique equilibrium.

Sensing Cost	$C_s \geq \frac{C_l}{2}$	$\frac{1-e^{-2C_l}}{4} \leq C_s \leq \frac{C_l}{2}$	
Sensing B_s^*	0	$B_s^{L*} \in \left[Ge^{-(2+C_l)}, Ge^{-2}\right]$	
Sensing Factor α	$0 \leq \alpha \leq 1$	$0 \leq \alpha \leq Ge^{-(2+C_l)}/B_s^{L*}$	$\alpha > Ge^{-(2+C_l)}/B_s^{L*}$
Leasing B_l^*	$Ge^{-(2+C_l)}$	$Ge^{-(2+C_l)} - B_s^{L*}\alpha$	0
Price π^*	$1 + C_l$	$1 + C_l$	$\ln\left(\frac{G}{B_s^{L*}\alpha}\right) - 1$
User k's SNR	$e^{(2+C_l)}$	$e^{(2+C_l)}$	$\frac{G}{B_s^{L*}\alpha}$
User k's Payoff	$g_k e^{-(2+C_l)}$	$g_k e^{-(2+C_l)}$	$g_k(B_s^{L*}\alpha/G)$

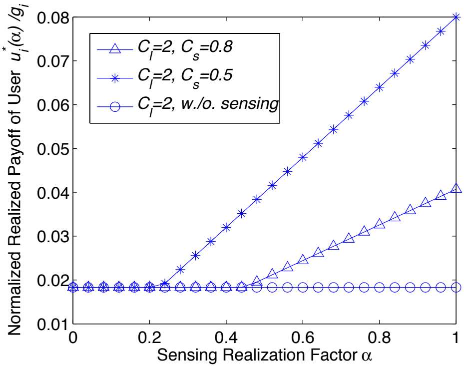
Impact of Sensing Uncertainty on Operator

- Realized profit increases with α
  - Can be smaller than no sensing
- Smaller C_s leads to more aggressive sensing and less reliable supply



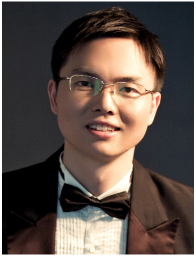
Impact of Sensing Uncertainty on Users

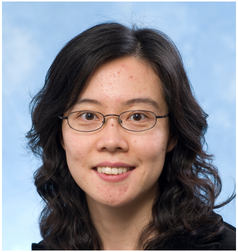
- Users' payoffs never decrease under sensing



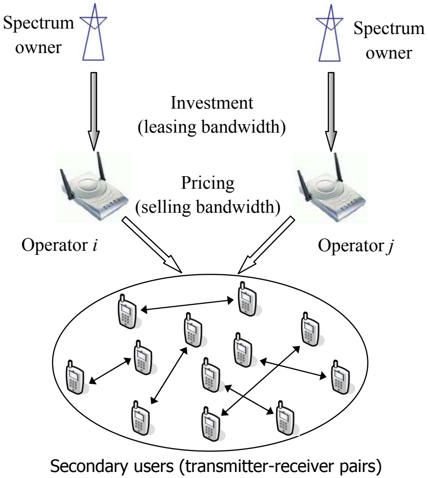
Spectrum Leasing Competition

L. Duan, J. Huang, and B. Shou, "Competition with Dynamic Spectrum Leasing," *IEEE Transactions on Mobile Computing*, 2013

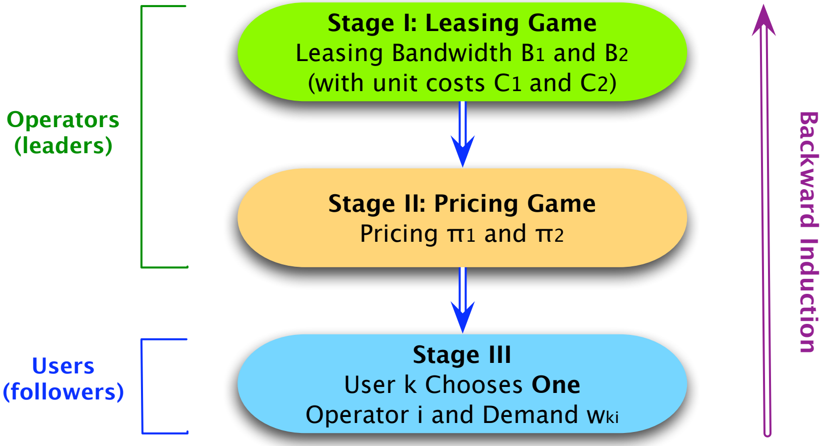




Network Model



Three-Stage Multi-leader-follower Game



Stage III: Users' Bandwidth Demands

- User k's payoff of choosing operator $i = 1, 2$

$$u_k(\pi_i, w_{ki}) = w_{ki} \ln\left(\frac{P_i^{\max} h_i}{n_0 w_{ki}}\right) - \pi_i w_{ki}$$

  - Optimal demand: $w_{ki}^*(\pi_i) = \arg\max_{w_{ki} \geq 0} u_k(\pi_i, w_{ki}) = g_k e^{-(1+\pi_i)}$
  - Optimal payoff: $u_k(\pi_i, w_{ki}^*(\pi_i))$

- User k prefers the "better" operator: $i^* = \arg\max_{i=1,2} u_k(\pi_i, w_{ki}^*(\pi_i))$

- Users demands may not be satisfied due to limited spectrum

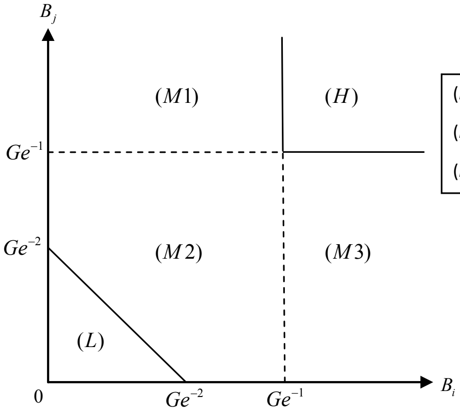
Stages II: Pricing Game

- Players: two operators
- Strategies: $\pi_i \geq 0, i = 1, 2$
- Payoffs: profit R_i for operator $i = 1, 2$:

$$R_i(B_i, B_j, \pi_i, \pi_j) = \pi_i Q_i(B_i, B_j, \pi_i, \pi_j) - B_i C_i$$

Stage II: Pricing Equilibrium

- Symmetric equilibrium: $\pi_1^* = \pi_2^*$.
- Threshold structure:
  - Unique positive equilibrium exists $B_1 + B_2 \leq Ge^{-2}$.



(L) : Unique nonzero equilibrium
($M1$)-($M3$) : No equilibrium
(H) : Unique zero equilibrium

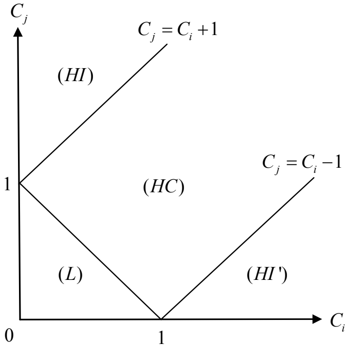
Stage I: Leasing Game

- Players: two operators
- Strategies: $B_i \in [0, \infty), i = 1, 2,$ and $B_1 + B_2 \leq Ge^{-2}$.
- Payoffs: profit R_i for operator $i = 1, 2$:

$$R_i(B_i, B_j) = B_i \left(\ln \left(\frac{G}{B_i + B_j} \right) - 1 - C_i \right)$$

Stage I: Leasing Equilibrium

- Linear in wireless characteristics $G = \sum_i g_i$;
- Threshold structure:
  - Low costs: infinitely many equilibria
  - High comparable costs: unique equilibrium
  - High incomparable costs: unique monopoly equlibrium



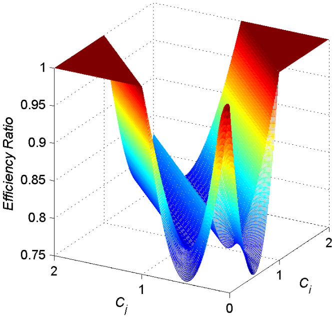
(*L*) : Infinitely many equilibria
(*HC*) : Unique equilibrium
(*HI*)-(*HI'*) : Unique equilibrium

Impact of Duopoly Competition on Operators

- Benchmark: Coordinated Case
  - Operators cooperate in investment and pricing to maximize total profit
- Define

$$\text{Efficiency Ratio} = \frac{\text{Total Profit in Competition Case}}{\text{Total Profit in Coordinated Case}}$$

- Can prove Price of Anarchy $= \min_{C_i, C_j}$ Efficiency Ratio$= 0.75$.



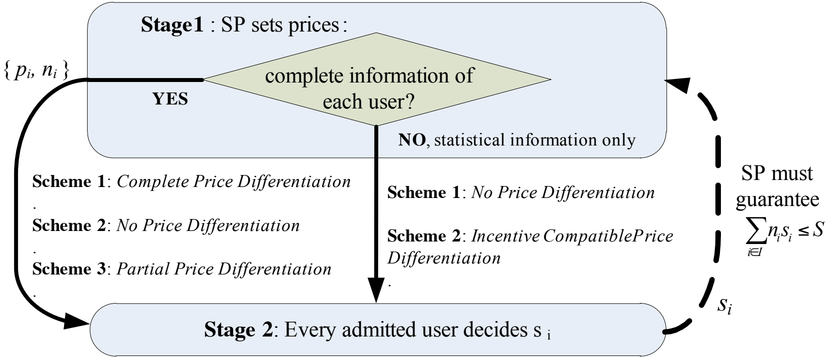
Partial Price Differentiation

S. Li and J. Huang, "Price Differentiation for Communication Networks," *IEEE/ACM Transactions on Networking*, 2013

Network Model

- One wireless service provider (SP)
- A set of $\mathcal{I}$ groups of users, where each group $i \in \mathcal{I}$ has
  - N_i homogenous users
  - Same utility function $u_i(s_i) = \theta_i \ln(1 + s_i)$
  - Groups have decreasing preference coefficients: $\theta_1 > \theta_2 > \cdots > \theta_I$
- The SP's decision for each group i
  - Admit $n_i \leq N_i$ users
  - Charge a unit price p_i (per unit of resource)
  - Subject to total resource limit: $\sum_i n_i s_i \leq S$

Two-Stage Stackelberg Game



- Analysis based on backward induction

Complete Price Differentiation: Stage II

- Each (admitted) group i user chooses s_i to maximize payoff

$$\underset{s_i \geq 0}{\text{maximize}}\ \theta_i \ln(1+s_i) - p_i s_i,$$

- The unique optimal demand is

$$s_i^*(p_i) = \max\left(\frac{\theta_i}{p_i} - 1, 0\right) = \left(\frac{\theta_i}{p_i} - 1\right)^+$$

Complete Price Differentiation: Stage I

- SP performs admission control $\mathbf{n}$ and determines prices $\mathbf{p}$:

$$\begin{aligned} \underset{\mathbf{n},\mathbf{p}\geq 0,\mathbf{s}\geq 0}{\text{maximize}} \quad & \sum_{i\in\mathcal{I}} n_i p_i s_i \\ \text{subject to} \quad & s_i = \left(\frac{\theta_i}{p_i} - 1\right)^+, \;\; i \in \mathcal{I}, \\ & n_i \in \{0, \ldots, N_i\} \;\; , \;\; i \in \mathcal{I}, \\ & \sum_{i\in\mathcal{I}} n_i s_i \leq S. \end{aligned}$$

  - The Stage II's user responses are incorporated

Complete Price Differentiation: Stage I

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$$
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& n_i \in \{0, \ldots, N_i\} \;\;, \;\; i \in \mathcal{I}, \\
& \sum_{i\in\mathcal{I}} n_i s_i \leq S.
\end{aligned}
$$

  - The Stage II's user responses are incorporated

- This problem is challenging to solve due to non-convex objectives, integer variables, and coupled constraint.

Complete Price Differentiation: Stage I

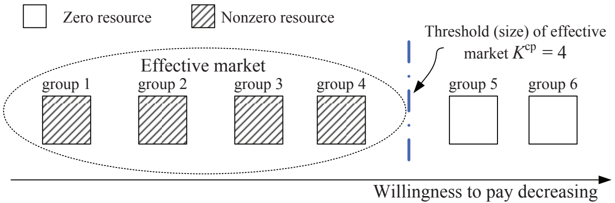
- The admission control and pricing can be decoupled
- At the unique optimal solution
  - Admit all users
  - Charge prices such that users perform voluntary admission control: there exists a group threshold K^{cp} and λ^{cp} with

$$p_i^* = \begin{cases} \sqrt{\theta_i \lambda^*}, & i \leq K^{cp}; \\ \theta_i, & i > K^{cp}. \end{cases}$$

and

$$s_i^* = \begin{cases} \sqrt{\frac{\theta_i}{\lambda^*}} - 1, & i \leq K^{cp}; \\ 0, & i > K^{cp}. \end{cases}$$

Complete Price Differentiation: Optimal Solution



- Effective market: includes groups receiving positive resources

Single Pricing (No Price Differentiation)

- Problem formulation similar as the complete price differentiation case
- Key difference: change the same price p to all groups
- Similar optimal solution structure
  - Effective market is no larger than the complete price differentiation case

Partial Price Differentiation

- The most general case
- SP can charge J prices to I groups, where $J \leq I$
  - Complete price differentiation: $J = I$
  - Single pricing: $J = 1$
- How to divide I groups into J clusters, and optimize the J prices?

Three-Level Decomposition

- Level I (Cluster Partition): partition I groups into J clusters
- Level II (Inter-Cluster Resource Allocation): allocate resources among clusters (subject to the total resource constraint)
- Level III (Intra-Cluster Pricing and Resource Allocation): optimize pricing and resource allocations within each cluster

Three-Level Decomposition

- Level I (Cluster Partition): partition I groups into J clusters
- Level II (Inter-Cluster Resource Allocation): allocate resources among clusters (subject to the total resource constraint)
- Level III (Intra-Cluster Pricing and Resource Allocation): optimize pricing and resource allocations within each cluster
- Solving Level II and Level III together is equivalent of solving a complete price differentiation problem

How to Perform Cluster Partition in Level I

- Naive exhaustive search leads to formidable complexity for Level I

Groups	$I = 10$		$I = 100$	$I = 1000$
Clusters	$J = 2$	$J = 3$	$J = 2$	$J = 2$
Combinations	511	9330	6.33825×10^{29}	5.35754×10^{300}

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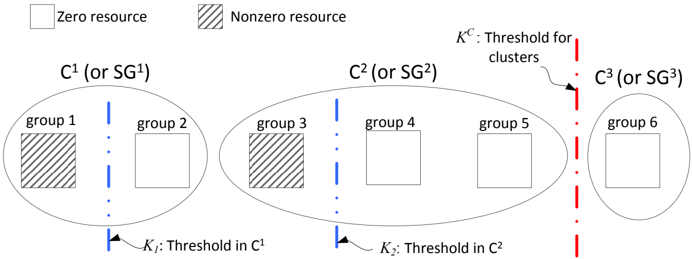
Combinations	511	9330	6.33825×10^{29}	5.35754×10^{300}

- Do we need to check all partitions?

Property of An Optimal Partition

- Will the following partition ever be optimal?



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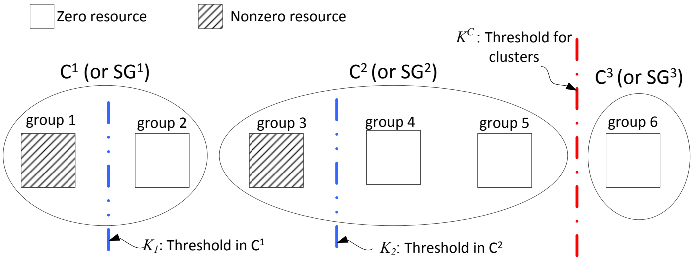

- No.

Property of An Optimal Partition

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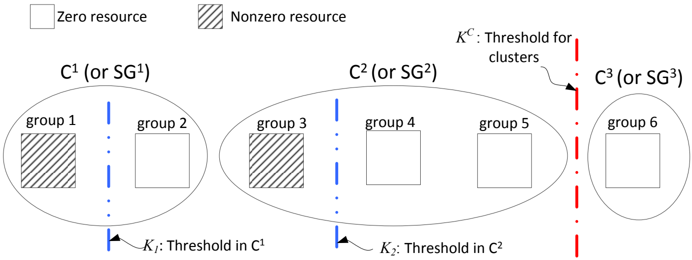


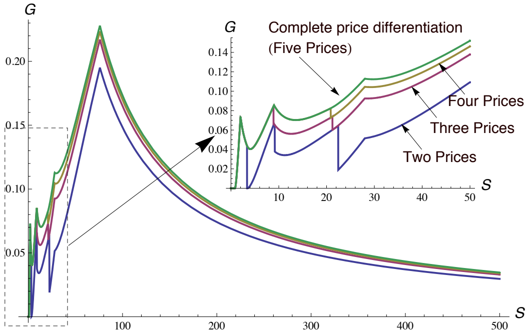
- No.
- We prove that group indices in the effective market are consecutive.

Reduced Complexity of Cluster Partition in Level I

Groups	$I = 10$		$I = 100$	$I = 1000$
Clusters	$J = 2$	$J = 3$	$J = 2$	$J = 2$
Combinations	511	9330	6.33825×10^{29}	5.35754×10^{300}
Reduced Combos	9	36	99	999

- The search complexity reduces to polynomial in I.

Relative Revenue Gain



- A total of $I = 5$ groups
- Plot the relative revenue gain of price differentiation vs. total resource
- Maximum gains in the small plot
  - $J = 3$ is the sweet spot

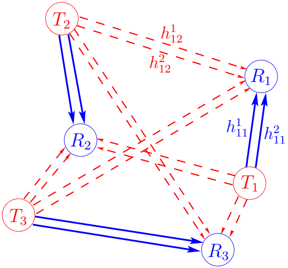
Distributed Power Control

J. Huang, R. Berry and M. Honig, "Distributed Interference Compensation for Wireless Networks," *IEEE Journal on Selected Areas in Communications*, 2006

Wireless Power Control

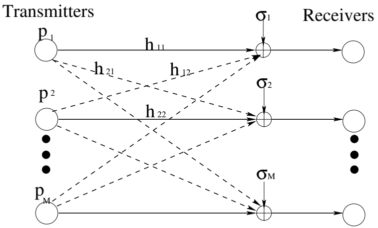
- Distributed power control in wireless ad hoc networks
- Elastic applications with no SINR targets
- Want to maximize the total network performance

Network Model



- Single-hop transmissions.
- A user = a transmitter/receiver pair.
- Transmit over multiple parallel channels.
- Interferences in the same channel.
- Our discussions focus on the single channel case.

Single Channel Communications



- A set of $\mathcal{N} = \{1, ..., n\}$ users.
- For each user $n \in \mathcal{N}$:
  - Power constraint: $p_n \in [P_n^{min}, P_n^{max}]$.
  - Received SINR (signal-to-interference plus noise ratio):

$$\gamma_n = \frac{p_n h_{n,n}}{\sigma_n + \sum_{m \neq n} p_m h_{n,m}}.$$

  - Utility function $U_n(\gamma_n)$: increasing, differentiable, strictly concave.

Network Utility Maximization (NUM) Problem

NUM

$$\max_{\{P_n^{\min} \le p_n \le P_n^{\max}, \forall n\}} \sum_n U_n(\gamma_n).$$

- Technical Challenges:
  - Coupled across users due to interferences.
  - Could be non-convex in power.
- We want: efficient and distributed algorithm, with limited information exchange and fast convergence.

Benchmark - No Information Exchange

- Each user picks power to maximize its own utility, given current interference and channel gain.
- Results in $p_n = P_n^{max}$ for all n.
  - Can be far from optimal.

Benchmark - No Information Exchange

- Each user picks power to maximize its own utility, given current interference and channel gain.
- Results in $p_n = P_n^{max}$ for all n.
  - Can be far from optimal.
- We propose algorithm with limited information exchange.
  - Have nice interpretation as distributed Pigovian taxation.
  - Analyze its behavior using supermodular game theory.

ADP Algorithm: Asynchronous Distributed Pricing

- Price Announcing: user n announces "price" (per unit interference):

$$\pi_n = \left|\frac{\partial U_n(\gamma_n)}{\partial I_n}\right| = \frac{\partial U_n(\gamma_n)}{\partial \gamma_n}\frac{\gamma_n^2}{p_n h_{n,n}}.$$

- Power Updating: user n updates power p_n to maximize surplus:

$$S_n = U_n(\gamma_n) - p_n \sum_{m \neq n} \pi_m h_{m,n}.$$

- Repeat two phases asynchronously across users.
- Scalable and distributed: only need to announce single price, and know limited channel gains ($h_{m,n}$).

ADP Algorithm

- Interpretation of prices: Pigovian taxation

ADP Algorithm

- Interpretation of prices: Pigovian taxation
- ADP algorithm: distributed discovery of Pigovian taxes
  - When does it converge?
  - What does it converge to?
  - Will it solve Problem **NUM**?
  - How fast does it converge?

Convergence

- Depends on the utility functions.

Convergence

- Depends on the utility functions.
- Coefficient of relative Risk Aversion (CRA) of $U(\gamma)$:

$$CRA(\gamma) = -\frac{\gamma U''(\gamma)}{U'(\gamma)}.$$

  - larger CRA $\Rightarrow$ "more concave" U.

Convergence

- Depends on the utility functions.
- Coefficient of relative Risk Aversion (CRA) of $U(\gamma)$:

$$CRA(\gamma) = -\frac{\gamma U''(\gamma)}{U'(\gamma)}.$$

  - larger CRA $\Rightarrow$ "more concave" U.

- **Theorem**: If each user n has a positive minimum transmission power and $CRA(\gamma_n) \in [1, 2]$, then there is a unique optimal solution of Problem **1-SC**, and the ADP algorithm globally converges to it.
- Proof: relating this algorithm to a fictitious supermodular game.

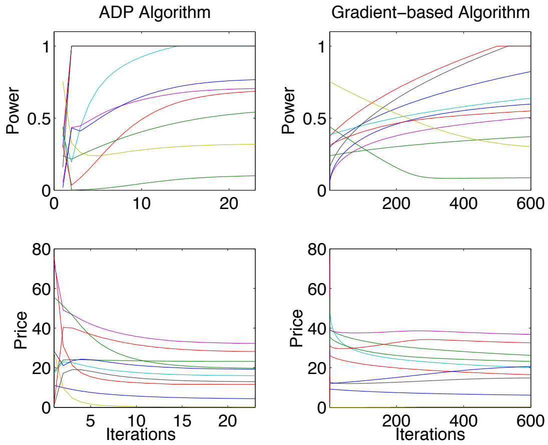
Supermodular Games

- A class of games with strategic complementaries
  - Strategy sets are compact subsets of $\mathbb{R}$; and each player's pay-off S_n has increasing differences:

$$\frac{\partial^2 S_n}{\partial x_n \partial x_m} > 0, \forall n, m.$$

- Key properties:
  - A PNE exists.
  - If the PNE is unique, then the asynchronous best response updates will globally converge to it.

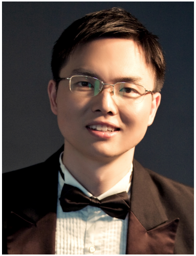
Convergence Speed



- 10 users, log utilities
- ADP algorithm converges much faster than a gradient-based method

Cellular Network Upgrade

L. Duan, J. Huang, and J. Walrand, "Economic Analysis of 4G Network Upgrade," *IEEE INFOCOM*, Turin, Italy, April 2013



When To Upgrade From 3G to 4G?

- Early upgrade:
  - More expensive, as cost decreases over time
  - Starts with few users, hence a small initial revenue
- Late upgrade:
  - Leads to a smaller market share
  - Delays 4G revenues
- Need a model that
  - Capture the above tradeoffs
  - Consider the dynamics of users adopting 4G and switching providers
  - Understand the upgrade timing between competing cellular providers

Duopoly Model

- Two competing operators
  - Initially both using 3G technology
  - Operator i decides to upgrade to 4G at time T_i
  - Each operator wants to maximize its long-term profit
- What will be the equilibrium of (T_1^*, T_2^*)?

Users Switching

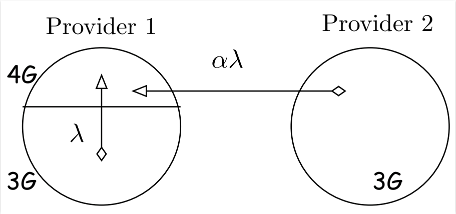
- W.L.O.G., assume $T_1 < T_2$
- Three time periods: $[0, T_1]$, $(T_1, T_2]$, and (T_2, ∞)

Users Switching

- W.L.O.G., assume $T_1 < T_2$
- Three time periods: $[0, T_1]$, $(T_1, T_2]$, and (T_2, ∞)
- When $t \in [0, T_1]$: No user switching.

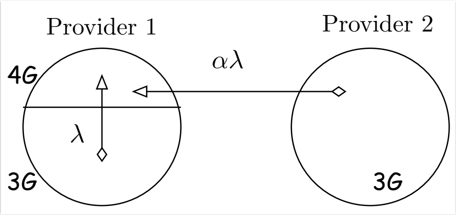
Users Switching

- When $t \in (T_1, T_2]$: both inter- and intra- operator user switching

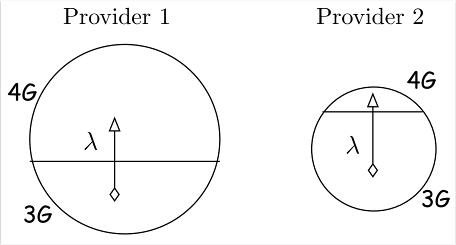


Users Switching

- When $t \in (T_1, T_2]$: both inter- and intra- operator user switching



- When $t \in (T_2, \infty)$: only intra-operator user switching



Network Value (Revenue)

- Network value depends on the number of subscribers
  - Assume that operator i has N_i 4G users, $i = 1, 2$
  - Total 4G network value is $(N_1 + N_2)\log(N_1 + N_2)$
  - Operator i's network value (revenue) is $N_i \log(N_1 + N_2)$

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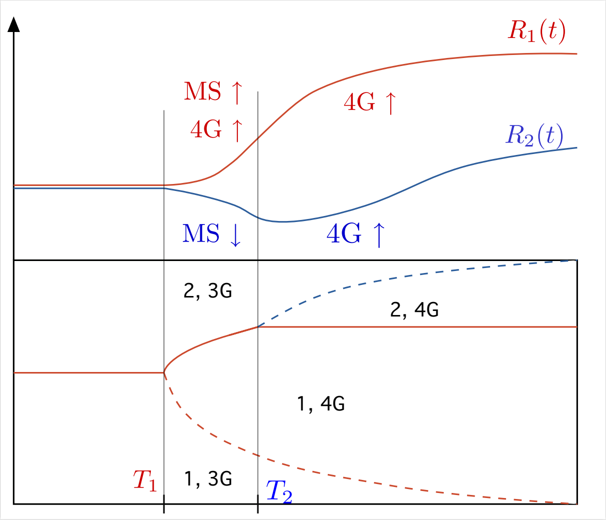
- Later upgrade $\Rightarrow$ take advantage of existing 4G population
- The revenue for 3G network is similar, with an coefficient $\gamma \in (0, 1)$

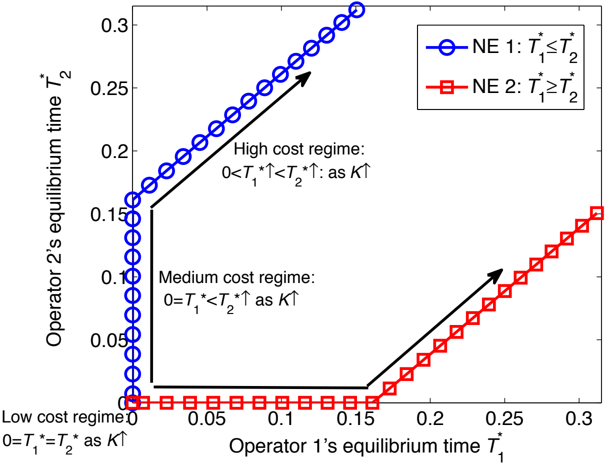
Revenue and Market Share



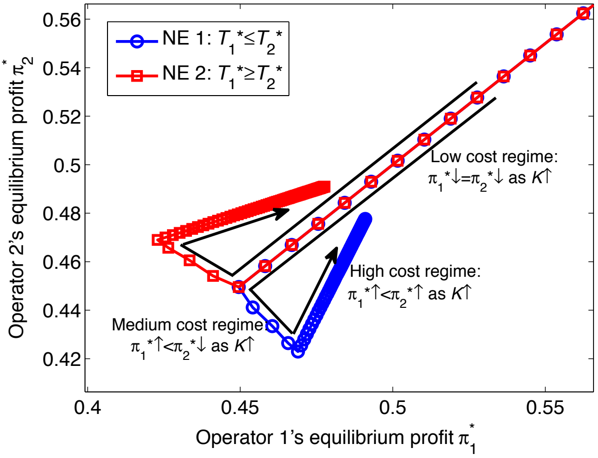
Upgrade Cost and Time Discount

- One-time upgrade cost:
  - K at time $t = 0$
  - Discounted over time: $K \exp(-Ut)$
- Revenue is also discounted over time by $\exp(-St)$
- Earlier upgrade $\Rightarrow$ larger revenue and larger cost

Equilibrium Timings



Equilibrium Profits



More Information: NCEL.ie.cuhk.edu.hk